Synopsis of the Ph.D. Thesis

Title: Confidence Intervals for Process Capability Indices With Respect to Random Effects Models and Autocorrelated Data

Research Scholar: JANE A. LUKE **Research Guide : Dr. K. K. JOSE**

Introduction

"Knowledge has to be improved, challenged and increased constantly or it vanishes", so observed Peter Drucker (1909-2005), the renowned management expert. It has been a never ending endeavour of man to improve on existing methods, processes and products. In the industrial context it has twin objectives - one represented by the manufacturers and the other by the consumers. While the former attempts to produce defect free products conforming to preset specifications, the latter seeks products which maximizes their satisfaction at the least cost. There have been efforts to reconcile this apparently conflicting interests. The much needed consensus is brought about by the concept of quality in production. It satisfies both the producer as his products register higher sale and the consumer as it derives him maximum satisfaction, propelling further demand for the products.

Every manufacturer strives to attain quality as it gives him an edge over his competitors. In achieving quality, the production process should be capable of churning out products conforming to preset specifications. Quality-improvement methods can be applied to any area within a company or organization, including manufacturing, process development, engineering design, finance and accounting and field service of products. Quality has become one of the most important consumer decision making factors in the selection among competing products and services.

The need for manufacturing defect-free products and continuously monitoring the production processes against preset quality barometers have led to the development of various process capability indices (PCIs) by industrial statisticians. Many statistical process control techniques have proven useful in quality and productivity improvements of products and processes. The objective of the research work is to formulate new methods for developing process capability indices capable of improving product quality which in turn raises customer satisfaction. In realization of the above, existing methods of computing confidence intervals for PCIs were examined. We have introduced multiple methods to choose a better supplier from among different suppliers. Further, we have derived confidence intervals for PCIs under one-way random effects model following Bissell's approximation.

Bell Telephone Laboratories and Western Electric Corporation are the pioneers who ushered in quality approach in industry. The first significant attempt to understand and remove process variation in a scientific way was made by Walter A. Shewhart of Bell Telephone Laboratories in the 1920's. He conducted his studies on understanding the various components of variation and that gave birth to the theory of Statistical Process Control (SPC). Shewhart's system of SPC was developed further and championed by his one-time colleague, W. Edwards Deming.

Shewhart developed the concept of statistical control chart in 1924. Harold F. Dodge and Harold G. Romig developed the concept of Acceptance Sampling towards the end of 1920's. By the middle of 1930's Statistical Quality Control (SQC) methods were widely in use. SQC is one of the most important applications of statistical techniques in industry. It is a set of statistical tools used by quality professionals. Statistical techniques in industry are mainly based on the theory of probability and sampling and are used in industries related to aircraft, textile, plastic, rubber, electrical equipment, telephone, transportation, chemicals and pharmaceuticals, medicine etc.

Process capability indices have been proposed to the manufacturing industry for measuring process production capability. PCIs are statistical devices used to measure the extent to which the process characteristic X under consideration meets specifications. As a fundamental technique in any production, quality and process improvement efforts, PCI is used to improve processes, products or services to achieve higher levels of customer satisfaction. A capability index is generally a function of the process parameters such as the mean μ, the standard deviation σ, the target value T, the lower specification limit L and upper specification limit U of X.

The concept of process capability indices C_p was first introduced by Juran et al. (1974). Juran had realized the need in industry for a single ratio or index to compare the specification interval with the actual process variation or spread. They defined the index as

$$C_p = \frac{allowable\ range\ of\ measurements}{actual\ range\ of\ measurements} = \frac{specification\ interval}{process\ spread} = \frac{U-L}{6\sigma}$$

The C_{pk} index was introduced to take care of the limitations of the C_p index, which assumes that the mean of the distribution is equal to the nominal size while estimating the proportion of defectives. A PCI that takes into account the changes in both the process mean μ and process variation σ, was felt necessary by many and that resulted in the proposal of the index C_{pk} by Kane (1986) and is defined as

$$C_{pk} = \frac{d - |\mu - M|}{3\sigma}$$

where $d = (U-L)/2$ and $M = (U+L)/2$. The other most frequently used process capability indices C_{pm} and C_{pmk} are given by

$$C_{pm} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}}$$

$$C_{pmk} = \frac{d - |\mu - M|}{3\sqrt{\sigma^2 + (\mu - T)^2}}$$

where L and U are as defined earlier, $d = \frac{U-L}{2}$, $M = \frac{U+L}{2}$ and T is the target value.

If the quality characteristic has a normal distribution, then Chou (1992) showed that $\widehat{C}_{pk}$ has a bivariate non-central t-distribution. Johnson and Kotz (1970) gave a simple asymptotic approximation to this non-central t-distribution. Using this approximation, the formula for the variance of $\widehat{C}_{pk}$ is given by

$$Var(\widehat{C}_{pk}) = \frac{1}{9n} + \frac{C_{pk}^2}{2n-2}$$

Based on this expression, Bissell (1990) proposed a $100(1-\alpha)\%$ lower confidence limit for C_{pk} as

$$B_{pk} = \widehat{C}_{pk} - Z_{1-\alpha}\sqrt{\frac{1}{9n} + \frac{\widehat{C}_{pk}^2}{2n-2}}$$

where $Z_{1-\alpha}$ is the $(1-\alpha)$ percentile value of the standard normal distribution.

The entire set of observations available on a process variable Y is said to have a one-way model if each of the observations can be assigned to one or the other of a certain number of groups or classes. In the context of observations collected from a production process, the groups/classes are called 'batches'. We would like to treat the one-way classified data in the random model set up, as the few batches we use in the process capability studies are only representatives of a large number of other possible batches. Such a data will be called a balanced data if the number of observations in each group are equal and otherwise unbalanced.

The mathematical model that describes the relationship between response and treatment for the one-way ANOVA is given by

$$y_{ij} = \mu + \alpha_i + e_{ij}$$

where y_{ij} represents the j^{th} observation$(j = 1, \cdots, n)$ on the i^{th} treatment $(i = 1, \cdots, b$ levels), μ is the common effect for the whole experiment, α_i represents the i^{th} treatment effect and e_{ij} represents the random error in the j^{th} observation on the i^{th} treatment. Assuming normal distribution, Bissell (1990) derived confidence intervals for the process capability indices. Following Bissell's approach, we derived confidence intervals for the process capability indices for the one-way balanced and unbalanced random effects model.

A process $\{X_t\}$ is said to be stationary Gaussian if it is stationary and Gaussian simultaneously. The first-order autoregressive process (AR(1)) given by Box et al. (1994) has the structure

$$X_t = \mu + \phi(X_{t-1} - \mu) + \xi_t$$

where X_t is the value of observation at time t, μ is the process mean and $\{\xi_t\}$ is a white noise process with zero mean and variance σ_ξ^2 where $\xi_t \sim N(0, \sigma_\xi^2)$. Also assume that $-1 < \phi < 1$.

Zhang (1998) derived the approximate expected value and variance of S^2, variances of $\widehat{C}_p$ and $\widehat{C}_{pk}$ in terms of the process autocorrelation function. Interval estimates of C_p and C_{pk} for autocorrelated processes are computed using the symmetrical construction method. Given the specification limits and a sample of process data $\{X_1, X_2, ..., X_n\}$, interval estimators of C_p and C_{pk} can be constructed as

$$\widehat{C}_p \pm k\hat{\sigma}_{C_p}, \quad \widehat{C}_{pk} \pm k\hat{\sigma}_{C_{pk}}$$

where k is a constant chosen by the user, $\hat{\sigma}_{C_p}$ and $\hat{\sigma}_{C_{pk}}$ are the sample standard deviations of $\widehat{C}_p$ and $\widehat{C}_{pk}$ respectively. For selected values of $\widehat{C}_p, \widehat{C}_{pk}, n$ and ϕ for AR(1) processes, Zhang (1998) investigated coverage probabilities for both indices.

Most of the traditional methods for assessing the capability of manufacturing processes are dealing with crisp quality. In quality control, such as other statistical problems, we may confront with imprecise concepts. One case is a situation in which specification limits (SLs) are imprecise. In this situation, fuzzy process capability indices (FPCI) are necessary for measuring the fuzzy quality in an in-control process. These FPCIs are helpful for comparing manufacturing processes with imprecise SLs. After the inception of the theory of fuzzy sets by Zadeh (1965), there have been efforts by many researchers to apply this idea in Statistics, quality control and optimization engineering techniques.

In a fuzzy process, let $U(a_u, b_u, c_u),\ L(a_l, b_l, c_l) \in F_T(\mathbb{R})$ be the fuzzy specification limits, where $a_u \geq c_l$. Then the fuzzy PCIs are defined as

$$\widetilde{C}_p = T\Big(\frac{a_u - c_l}{6\sigma}, \frac{b_u - b_l}{6\sigma}, \frac{c_u - a_l}{6\sigma}\Big)$$

$$\widetilde{C}_{pk} = T\Big(\frac{a_u - c_l - 2|\mu - m|}{6\sigma}, \frac{b_u - b_l - 2|\mu - m|}{6\sigma}, \frac{c_u - a_l - 2|\mu - m|}{6\sigma}\Big)$$

and

$$\widetilde{C}_{pm} = T\Big(\frac{a_u - c_l}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{b_u - b_l}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{c_u - a_l}{6\sqrt{\sigma^2 + (\mu - T)^2}}\Big)$$

where $m = (b_u + b_l)/2$ and T is the target value.

Review of Literature

The use of process capability indices to quantify process performance within its manufacturing facilities were initiated in the US during the early 1980's, when Ford Motor Company began using C_p and C_{pk} to track performance, see Kane (1986). By 1991, all of the big US automakers were aggressively using SPC and capability indices to monitor and improve product quality. Kane (1986) reported that other companies also required capability standards for purchasing decisions.

To assess the reliability of PCIs, lower confidence limits are very often used and several approximate lower confidence limits have been developed over the years. The formal practice of proposing confidence intervals for PCIs began with Chou et al. (1990) when they prepared elaborate tables giving lower confidence limits for various indices. Since then, there have been numerous attempts to propose approximate confidence intervals for PCIs by Heavlin (1988), Bissell (1990), Kushler and Hurley (1992), Nagata and Nagahata (1994), Franklin and Wasserman (1991, 1992), Pearn et al. (2004), Wu and Pearn (2005) and Lin et al. (2005). Pearn and Liao (2006) obtained approximate confidence intervals when the quality characteristic of interest followed a univariate normal distribution. Detailed review of PCIs along with extensive bibliographies are available in Kotz and Johnson (1993) and Kotz and Lovelace (1998).

Bootstrapping is a newer, non-parametric, but computer-intensive statistical technique, introduced by Efron (1979) that plays an increasingly important role in modern statistical analysis and applications. Bootstrap iteration has been particularly useful in improving the accuracy of confidence intervals. Calibration of the bootstrap was first introduced by Hall (1986). Franklin and Wasserman (1991) introduced the use of bootstrap sampling procedures for deriving non-parametric confidence intervals for the process capability index C_{pk}.

The generalized p-value approach for hypothesis testing have been introduced by Tsui and Weerahandi (1989) and the generalized confidence intervals (GCI) by Weerahandi (1993). Together, these are referred to as the generalized variable approach or generalized inference procedure. The concepts of generalized p-values and generalized confidence intervals have turned out to be extremely fruitful in obtaining tests and confidence intervals for some complex problems where standard procedures are difficult to apply. For detailed discussion and numerous applications, see Weerahandi (1995, 2004). The areas of application included comparison of means, testing and estimation of parametric functions of normal and other distributions by Tsui and Weerahandi (1989) and Krishnamoorthy and Mathew (2003).

A study on measuring process yield based on the capability index C_{pm} was done by Pearn and Lin (2004). Assuming normal distribution, Mathew et al. (2007) developed the generalized confidence intervals methodology for constructing confidence limits for process capability indices. Under the one-way random model, Kurian et al. (2008) proposed the GCI methodology in the construction of confidence limits for process capability indices both in balanced and unbalanced environment. Using generalized confidence intervals for the process capability index C_{pm}, Hsu et al. (2008) offered a method to assess the minimum performance of one manufacturing process/one supplier and also to assess the smallest performance of several manufacturing processes/several suppliers for equal as well as unequal process variance.

With the development of measurement technology and data acquisition technology, sampling frequency is getting higher and the existence of autocorrelation cannot be ignored. Shore (1997) investigated the effect of autocorrelations on PCIs and modelled the autocorrelation structure of a set of data using an autoregressive model of order three (AR(3)). Zhang (1998) discussed the use of the process capability indices C_p and C_{pk} when the process data were autocorrelated. Interval estimation procedures for C_p and C_{pk} were proposed and their properties were also studied by them. Process capability analysis when observations autocorrelated was addressed using time series modelling and regression analysis by Noorosana (2002).

The comparison of process capability indices C_p, C_{pk}, C_{pm} and C_{pmk} when data autocorrelated were done by Guevara and Vargas (2007). Variances for their estimators were derived and coverage probabilities of confidence intervals computed. Lovelace et al. (2009) developed lower confidence limits for PCIs C_p and C_{pk} when data are autocorrelated. Sun et al. (2010) analyzed five estimation schemes of process capability for autocorrelated data and their comparisons discussed for small sample and large sample data. Properties of C_p and C_{pk} for autocorrelated data in the presence of random measurement errors were explained in detail by Scagliarini (2002, 2010).

Lee (2001) proposed a model to compute the fuzzy $\widetilde{C}_{pk}$ as a PCI when observations were fuzzy numbers, by concentrating on the construction of the membership function of the FPCI. Parchami et al. (2005) introduced fuzzy PCIs determining the relations between PCIs when specification limits are fuzzy numbers. Parchami and Mashinchi (2007) proposed a Buckley approach based algorithm to determine fuzzy estimates (which contains both point and interval estimates) for PCIs providing more information for the practitioners. Kaya and Kahraman (2007) proposed a methodology for air pollution control by using fuzzy and traditional PCIs. Again, fuzzy robust PCIs for a piston manufacturing company were explained in Kaya and Kahraman (2010).

Objectives of the Present Work

Exploring new methods for developing process capability indices with a view to improving the product quality and enhancing customer satisfaction was an objective of this research work. In realization of the above, existing methods of computing confidence intervals for PCIs were examined. There remained some inadequacies in the hitherto known methods. The effort was to improve and bringforth a more reliable and realistic methods for computing confidence intervals for PCIs using multiple methods.

The main objectives of the research work undertaken are as follows.

1. To derive confidence intervals for the difference between two PCIs for normal distribution and one-way random effects ANOVA model using the generalized confidence intervals methodology.

2. To improve the accuracy of the generalized lower confidence limit for PCIs using bootstrap calibration.

3. To study the lower confidence limit for the process capability index C_{pk} under a one-way random effects model with balanced and unbalanced data following the Bissell's approximation method.

4. To compare the performance of the newly proposed confidence limit for PCIs for one-way random effects model with the generalized lower confidence limit method and standard bootstrap method using simulation studies and data sets.

5. To compute confidence intervals for PCIs when observations are autocorrelated.

6. To introduce approximate confidence limits for the PCI C_{pk} using Bissell (1990), Nagata and Nagahata (1994), Heavlin (1988) and Kushler and Hurley's(1992) methods when observations follow an AR(1) model.

7. To apply fuzzy techniques for constructing confidence intervals for fuzzy process capability indices.

Summary of the Work

The thesis explores confidence intervals for PCIs and difference between two PCIs under several models. It is structured in 7 chapters. Chapter 1 gives an introduction to the topic of study as well as a brief review of literature along with summary of the work done.

Chen and Tong (2003) have constructed a bias corrected percentile bootstrap confidence intervals for $C_{pk1} - C_{pk2}$ when observations follow normal distribution. The coverage probabilities are poor in many cases. In Chapter 2, we discuss confidence intervals for the difference between PCIs for two processes derived by GCI method. The indices C_{pk}, C_{pmk} and C_{pm} are considered in this study. The performance of the proposed method is assessed using simulation procedures. The results are also demonstrated using an example drawn from industrial contexts. The study helps to make a wise choice between two suppliers when observations are normally distributed. Further, we made comparison between GCI and standard bootstrap method for confidence intervals for PCIs and the same appeared in Jane and Jose (2011a).

Chapter 3 explores the difference between process capability indices for two processes under one-way random effects model using GCI method. The method provides coverage probability close to the nominal value in almost all cases as shown via simulation and data set. The method assists the manufacturer in selecting a better supplier and the paper published in Jose and Jane (2011b).

Confidence intervals for process capability indices using GCI method was introduced by Kurian et al. (2008). Their methodology resulted in inaccurate confidence intervals in many cases, the coverages could be unsatisfactory for certain sample size and parameter combinations. Chapter 4 presents simple and accurate methods for deriving generalized confidence intervals for PCIs via bootstrap calibration, under one-way random model with balanced data. Coverage probabilities and expected values are computed and compared, both before and after calibration. It has been noted that the lower confidence limits can be unsatisfactory before calibration, in terms being too conservative. GCI exhibited significant improvement after calibration. The study has been illustrated with the help of industrial examples.

Further, it provides the lower confidence limits for C_{pk} for the one-way random effects model with balanced data using Satterthwaite's method following Bissell's approximation formula. Its coverage probabilities and expected values are calculated. Again coverage probabilities and expected values are computed using standard bootstrap (SB) method. These two are compared using simulation study and real data sets. The results reveal that the proposed confidence limit gives higher expected values than the existing methods and coverage probability close to the nominal value. The results are brought about in Jose and Jane (2012a).

Chapter 5 describes lower confidence limit for C_{pk} for the unbalanced one-way random effects model following Bissell's approximation method. The proposed limit is compared with the generalized confidence limit obtained using GCI method. To assess the accuracy of the method simulation study is presented and the results are illustrated with an industrial example. The study is published in Jose and Jane (2012b).

Zhang (1998) introduced the use of the process capability indices C_p and C_{pk} when the process data are autocorrelated. The interval estimation procedures for C_p and C_{pk} was proposed by them and their properties were also studied. Guevara and Vargas (2007) dealt with the comparison of process capability indices C_p, C_{pk}, C_{pm} and C_{pmk} when data were autocorrelated. Chapter 6 discusses the effect of autocorrelations on various process capability indices. Confidence intervals are constructed for PCIs when data are both independent and autocorrelated. Approximate lower confidence limits for various C_{pk} are computed for AR(1) model. Simulation studies and industrial examples are considered to demonstrate the results.

Chapter 7 focusses on fuzzy PCIs. In some cases specification limits are not precise numbers and they are expressed in fuzzy terms. Using the theory of fuzzy sets, fuzzy PCIs are developed to determine whether a production process is capable of producing items within the fuzzy specification limits. Computation of confidence intervals for PCIs for various C_{pk} is done through fuzzy approach. Numerical examples are given to illustrate the method.

Thus the thesis deals with deriving the lower confidence limits for process capability indices under one-way random effects model both in balanced and unbalanced environment and for autocorrelated data. It has many applications in industrial sector, business field, time series modelling and production scenario. The results of the study have been accepted by or communicated to leading International and National journals. The same have been presented both in many International and National Conferences.

Papers Accepted by/Communicated to Journals

1. Jane A. Luke and K. K. Jose (2006). Exponentially Weighted Moving Average Control Chart and Autoregressive Processes, *STARS: International Journal*, **7(1)**, 23-37.

2. Jose, K. K. and Jane A. Luke (2010). Generalized Confidence Intervals for Variance Components in Two-Way Random Effect model with Balanced Data, *Proceedings of UGC and KSCSTE Sponsored Seminar on New Trends in Applied Statistical Methodology*, 59-72.

3. K. K. Jose and Jane A. Luke (2012). Confidence Intervals for Process Capability Indices for the Unbalanced One-Way Random Effect ANOVA Model, *Quality and Reliability Engineering International*, **28**, 371-375.

4. K. K. Jose and Jane A. Luke (2011). Comparing Two Process Capability Indices Under Balanced One-Way Random Effect Model, *Quality and Reliability Engineering International*, DOI: 10.1002/qre.1297, Wiley Online Library, John Wiley & Sons Ltd.

5. Jane A. Luke and K. K. Jose (2011). Confidence Intervals for Process Capability Index C_{pk}, *Proceedings of the UGC-Sponsored National Seminar on Discrete Mathematics & Computational Statistics*, 41-53.

6. K. K. Jose and Jane A. Luke (2012). On Confidence Intervals for Process Capability Indices in One-Way Random Model, *Communications in Statistics-Simulation and Computation*, **41(10)**, 1805-1815.

7. K. K. Jose and Jane A. Luke (2012). Comparison Between Two Process Capability Indices Using Generalized Confidence Intervals, *International Journal of Advanced Manufacturing Technology*, (communicated-under revision).

8. Jane A. Luke and K. K. Jose (2012). Confidence Intervals for Process Capability Indices When Data Exhibits Autocorrelation, *Journal of Applied Statistics*, (communicated-under revision).

9. Jane A. Luke and K. K. Jose (2012). Fuzzy Process Capability Indices, *Proceedings of National Seminar on Recent Trends in Fuzzy Mathematics and its Applications*, (in press).