Synopsis of the Ph.D. Thesis

Title: Modeling and Analysis of Queueing systems with Discrete Autoregressive Arrivals and Counting Processes

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1 Introduction

This thesis is mainly intended to study various discrete time multiserver queueing systems governed by a discrete autoregressive process of order 1 with a particular marginal distribution. In this integer valued time queue, the time is divided into slots of equal size and one slot is needed to serve a packet by a server. We assume that packet arrivals occur at the beginning of the slots and departures occur at the end of the slots. Here $\{X(t) : t = 0, 1, 2, ...\}$ represents packet arrivals so that $X(t)$ be the number of packets arriving at the beginning of the t th slot. Let $\{N(t) : t = 0, 1, 2, ...\}$ be the number of packets in the system say system size , immediately before arrivals at the beginning of the tth slot. For this queueing system the stationary distribution of the system size and the waiting time distribution of an arbitrary packet are obtained with the help of matrix analytic methods and Markov regenerative theory. Multiserver queues are often encountered in telecommunications systems and have special importance in the design of asynchronous transfer mode or ATM network. These queues are directly applicable to intelligent transportation systems, call centers, networks, telecommunications, server queueing, mainframe computer of telecommunications terminals, advanced telecommunications systems, and traffic flows.

ATM is a packet switching protocol that encodes data into small fixed sized cells and the goal of this technology is to design a single networking strategy that could transport real time video conference and audio as well as image files, text and email. This differs from other technologies based on packet -switched networks such as the Internet Protocol or Ethernet, in which variable sized packets are used. ATM exposes properties from both circuit switched and small packet switched networking, making it suitable for wide area data networking as well as real time media transport.

Markovian arrival processes (MAP) are constructed from exponential distribution for eg. Markov modulated Poisson process, can be used in classical teletraffic theory, where the traffic exhibits high autocorrelation. Many authors modeled correlated arrival processes as MAPs which generalize Markov Modulated Poisson Process (MMPPs) in a continuous time frame work and Markov Modulated Bernoulli Process (MMBPs) in discrete frame work. Due to the memoryless property of the exponential distribution, the analysis of teletraffic models is usually tractable. But these models are over parameterized to capture both the marginal density and the correlation structure of measured data accurately. Also the model identification and estimation become a major task with these models. Usually in order to reduce the number of parameters, we use the two state MMPPs or MMBPs simply because they have only four parameters or fewer to estimate and their autocorrelations are exponentially or geometrically decaying, which is one of the salient features of the traffic in the telecommunication networks such as ATM.

The time series models with relatively very few number of parameters are well suited for accurate and meaningful modeling of various traffic sources in high speed applications. Among time series, the discrete autoregressive process of order 1 (DAR [1]) is a Markov process with geometrically decaying autocorrelation function, which can exhibit general distribution. Also DAR [1] is much simpler than the BMAP (Batch Markovian arrival process) which can also accomodate general distribution. Thus DAR(1) can be considered as a good candidate to model input traffic in telecommunication networks such as ATM network. The first-order discrete autoregressive equation can be in the form,

$$X(t) = (1 - Z(t))X(t-1) + Z(t)Y(t), t = 0, 1, 2, ... \quad (1)$$

where $\{Z(t) : t = 0, 1, 2, ...\}$ are i.i.d. binary r.v.s with $P[Z(t) = 0] = \beta(0 \leq \beta < 1)$ and $P[Z(t) = 1] = 1 - \beta$. $\{Y(t) : t = 0, 1, 2, ...\}$ is a sequence of i.i.d random variables. $\{Z(t) : t = 0, 1, 2, ...\}$ is assumed to be independent of $\{Y(t) : t = 0, 1, 2, ...\}$. $Y(t)$ assumes a marginal distribution $b(x)$, where $b(x) = P[Y(t) = x]$. DAR(1) is determined by the parameter β and the distribution $\{b(x) : x = 0, 1, 2, ...\}$ of $Y(t)$.

Usually generating function and Laplace transform methods are used to find the steady state probabilities of the queueing systems such as $M/M/1$, $M/M/\infty$, $G/G/1$ etc. But the numerical tractability of queueing systems through these methods become complicated when we assume non exponential interarrival or service time distribution.The introduction of matrix analytic methods in solving queueing problems reduced the problem of numerical intractability considerably and increased the implementation of queueing models to analyze practical situations taking non exponential interarrival and service time distribution which are more suitable for practical applications. G/M/1 and M/G/1 Markov chains model systems are deployed in modeling modern communication systems. Each one of the queue exhibits characteristics that involve one of the matrix solutions. G/M/1 exhibits geometric properties enabling matrix geometric techniques while M/G/1 systems are harder to solve and involve matrix analytic techniques. For details see Neuts [1989].

Queues with negative arrivals are used as a control mechanism in many telecommunication and computer networks, neural networks, manufacturing systems etc. In a computer network or a database, negative customers can represent viruses or commands to delete some transaction. In a manufacturing system, negative customers can represent orders of demand and in neural networks a positive customer is interpreted as excitation while a negative one as inhibition.

A negative arrival does not receive service and has the effect of removing a customer from the queue. If it arrives when the queue is empty, it has no effect on the system and it is lost. For a fixed number k, when a negative arrival of size $l(< k)$ arrives at the system, it removes min $(l; N(t))$ customers in the queue. However, when a negative arrival of size $l(\geq k)$ arrives at the system, it removes min $(k, N(t))$ customers in the queue. Positive customers are ordinary ones who form a queue but the negative customers do not form a queue. Negative customer is a signal to delete a positive customer in the system if any presents, and disappeared immediately. The negative arrivals makes the system less congested than if they were not present.

Another problem of main concern in this thesis is to develop more general types of count processes. The widespread popularity of the Poisson model for count data arises from its simple derivation as the number of arrivals in a given time period assuming exponentially distributed interarrival times. The Poisson count model is valid only in the case where the data of interest support the restrictive assumption of equi-dispersion, that is the conditional variance equals the conditional mean, but typically the conditional variance exceeds the conditional mean(over-dispersion). There are also cases where the conditional mean exceeds the conditional variance (under-dispersion). In either case, the estimation based on Poisson model is inefficient and leads to biased inference. In order to overcome this limitations some generalized models for count data based upon Mittag Leffler as well as Gumbel interarrival process are also developed in this thesis. These new count models can model over as well as under and equidispersed real data sets, especially on climate data. Growing concerns over the effects of climate change and environmental issues have received current interest due to the influence of climate on disease dynamics and agricultural crop production. These count models can be used to analyze the relationship between climate change and epidemic incidence as well as economic impacts.

The Poisson count model becomes inadequate in most of the econometric applications. Hence we assume that the waiting times between the events are independent but not exponential. Instead they are assumed to follow some other distribution with a nonconstant hazard function. If the hazard function is a decreasing function of time, the distribution displays negative duration dependence. If the hazard function is an increasing function of time, the distribution displays positive duration dependence. There is a link between duration dependence and dispersion. It is shown that negative duration dependence (asymptotically) causes over-dispersion and positive duration dependence causes under-dispersion. In both cases, the conditional probability of a current occurrence depends on the time since the last occurrence rather than on the number of previous events.

A count model can be derived based upon the relationship between interarrival times and their count model equivalent. Let Y_n be the time from the measurement origin at which the n^{th}event occurs. Let $X(t)$ denote the number of events that have occurred upto time t. The relationship between interarrival times and the number of events is

$$Y_n \leq t \quad \Leftrightarrow \quad X(t) \geq n$$

$$\begin{aligned} M_n(t) = P[X(t) = n] &= P[X(t) \geq n] - P[X(t) \geq n+1] \\ &= P[Y_n \leq t] - P[Y_{n+1} \leq t] \end{aligned}$$

Assume the c.d.f. of Y_n as $F_n(t)$, then

$$M_n(t) = P[X(t) = n] = F_n(t) - F_{n+1}(t) \tag{2}$$

2 Review of literature

Recently much research attention has been focussed on methods for providing video services using ATM networks. Several fundamental issues regarding the transmission of video over ATM networks remain unresolved. Heyman et al. [1992] statistically analyzed and simulated Video Teleconference Traffic in ATM networks. Again Heyman et al. [1994] modeled the Video Teleconference Traffic from VBR video coders. The discrete autoregressive process of order 1 [DAR(1)] with non Gaussian marginal is known to be a good model for VBR coded teleconference traffic as in Elwalid et al. [1995].

Hwang et al. [2002] obtained the waiting time distribution of the discrete time single server queue with DAR(1) input. Again Hwang and Sohraby [2003] obtained the closed form expression for the stationary probability generating function of the system size of the discrete time single server queue with DAR(1) input. Hwang et al. [2004] analyzed the queueing behavior of multiple first-order autoregressive sources. Choi and Kim [2004] analyzed a multiserver queue fed by DAR(1) input. Kamoun [2006] analyzed the discrete-time queue with autoregressive inputs revisited.

A queueing system with discrete autoregressive arrivals is analyzed by Kim et al. [2007]. Mean queue size in a queue with discrete autoregressive arrivals of order p is obtained by Kim et al. [2008]. Analytic approximations of queues with lightly- and heavily-correlated autoregressive service times is discussed in Dieter et al. [2011].

The quasi-negative binomial distribution (QNBD) was introduced by Janardan [1975] and Sen and Jain [1996]. But unfortunately the moments of this distribution appear in an infinite series which is not suitable for summation. Hence Ahmad et al. [2010] introduced a new model called quasi-negative-binomial distribution -II(QNBD-II).

Gelenbe [1991] introduced the concept of negative customers in queues. Artalejo [2000] obtained the networks with positive and negative arrivals, called G- networks. Many continuous-time queueing models with negative arrivals have been discussed during the last years, but the analysis of discrete-time queueing models has received considerable attention in the scientific literature over the past years in view of its applicability in the study of many computer and communication systems in which time is slotted, for instance ATM and BISDN .

Glynn et al. [1991] analyzed queues with negative customers. Shin [2003] described a queue with positive and negative arrivals governed by a Markov chain. The work about negative customers in discrete-time can be found in Atencia [2004] and Moreno [2005] where the authors considered the single-server discrete-time queue with negative arrivals and various killing disciplines caused by the negative customers. Jinting et al. [2009] considered a discrete-time retrial queue with negative customers and unreliable server. A discrete time queueing system with negative customers and single working vacation is analyzed by Songfang and Chen in [2011].

Inusah and Kozubowski [2006] introduced the discrete skew Laplace random variable. DSL distribution is useful in answering questions on whether the positive and negative arrivals have the same geometric distribution. Seethalakshmi and Jose [2008] studied autoregressive processes with discrete skew Laplace as marginal distribution and various properties are explored. Bouzar [2004] discussed discrete semi-stable distribution and discrete geometric semi-stable distributions. Jayakumar and Bouzar [2006] proposed several stationary INAR(1) models with discrete semistable marginals and related distributions.

Pillai [1990] introduced the Mittag-Leffler distribution in terms of Mittag-Leffler functions. Pillai and Jayakumar [1995] introduced a new class of discrete distributions , namely discrete Mittag-Leffler distributions (DML)which is a generalization of geometric distribution and developed a first order autoregressive process with discrete Mittag-Leffler marginal distribution.

The Poisson process can be taken as a sequence of independently and identically exponentially distributed waiting times. To derive a generalized model we replace the exponential distribution with a less restrictive non negative distribution. Possible models known from the duration literature are the Weibull see McShane et al. [2008], the gamma(including generalized gamma) see Winkelmannn [1995], and the log normal distributions see Bradlow et al. [2002], Everson and Bradlow [2002], Miller et al. [2006]. Both Weibull and gamma nest the exponential distribution and both allow for a monotone hazard rate function that is duration dependent. Recently Jose and Abraham [2011] introduced a new count model with Mittag-Leffler interarrival times.

The Gumbel distribution was first developed by a German mathematician Emil Gumbel [1891-1966]. Gumbel's focus was primarily on applications of extreme value theory to engineering problems, in particular modeling of meteorological phenomena such as annual flood flows. Jose and Bindu [2012] introduced a count model with Gumbel interarrival times for modeling climate change.

3 Objectives of the Present Work

The present work is concerned with a detailed study of various multiserver queues in which the arrival process is governed by a discrete autoregressive process of order 1 [DAR(1)] with a particular marginal distribution. New generalized counting processes with Mittag-Leffler as well as Gumbel inter-arrival time distribution are also introduced in the study . The present study has been undertaken with the following specific objectives:

[1] To study various discrete time multiserver queues fed by DAR(1) process with Quasi-Negative Binomial Distribution-II, Discrete Skew Laplace distribution, Discrete Stable distribution, and Discrete Mittag-Leffler distribution as marginal.

[2] To obtain the closed form expression for the stationary distribution of the system size of the queue with the help of matrix analytic methods and Markov regenerative theory.

[3] To derive the waiting time distribution of an arbitrary packet.

[4] To illustrate the effect of autocorrelation coefficient and parameters of the input traffic on the stationary distribution of system size and waiting time empirically.

[5] To validate it with respect to real data sets.

[6] To derive new generalized counting processes with Mittag-Leffler as well as Gumbel interarrival time distributions.

[7] To simulate these new count models by Markov Chain Monte-Carlo(MCMC) methods, using Metropolis-Hastings algorithm.

[8] To derive the features of the new counting models.

[9] To apply the model to real data sets on interarrival times of customers in a bank counter.

[10] To study the relationship between climate change and epidemic incidences and apply the model with respect to real data sets.

4 Summary of the Present Work

The thesis is organized into 7 chapters. Chapter 1 covers an introduction to the topic of study as well as a brief review of literature. In chapter 2, analysis of a multiserver queue in which the arrival process is governed by a discrete autoregressive process of order 1 [DAR(1)] with Quasi-Negative Binomial Distribution-II as the marginal distribution is given. Simulation study of the sample path of the arrival process is conducted. For this queueing system the stationary distribution of the system size and the waiting time distribution of an arbitrary packet is obtained with the help of matrix analytic methods and Markov regenerative theory. Negative binomial distribution, generalized Poisson distribution, Borel-Tanner distribution and zero truncated generalized Poisson distribution are taken as special cases of Quasi-Negative Binomial Distribution-II. The simulation and empirical study of the effect of autocorrelation function of input traffic on the stationary distribution of the system size as well as waiting time of an arbitrary packet is done through computer programmes. The model is applied to a real data regarding the number of customers waiting for checkout in an airport and it is established that the model well suits this data.

Computer systems without viruses have been modeled and analyzed by using conventional queueing models. However, conventional queueing models are not appropriate to the system suffering from virus attacks since the effects of viruses to the system are different from those of ordinary jobs. In Chapter 3 analysis of a multiserver queue with s servers $(s > 0)$ having constant service rate in which the input is ATM multiplexer with VBR coded teleconference traffic with both positive and negative arrivals is done. The negative arrivals are assumed to be caused by virus attacks. The input traffic of the queue is assumed to be DAR(1) with discrete skew Laplace distribution as the marginal distribution. Simulation study of the sample path of the arrival process is conducted. The stationary distributions of the system size and the waiting time of an arbitrary packet is obtained by using matrix analytic methods and the Markov regenerative theory. The quantitative effect of the stationary distribution of system size and waiting time on the autocorrelation function as well as the parameters of the input traffic is illustrated numerically. The model is applied to a real data with positive and negative arrivals and it is established that the model well suits this data.

Various INAR(1) models with stable, semistable and related distributions as marginal distribution are introduced in chapter 4 which can be used as the arrival pattern of discrete time queues. Especially DAR(1)/D/s queue with discrete Stable distribution as the marginal distribution is analyzed in this chapter. Simulation study of the sample path of the arrival process is conducted. Based on the matrix analytic methods and by using the Markov regenerative theory, the stationary distributions of the system size and the waiting time of an arbitrary packet is obtained. The quantitative effect of the stationary distribution of system size and waiting time on the autocorrelation function as well as the parameters of the input traffic is illustrated empirically. The model is applied to a real data and it is established that the model well suits this data.

Analysis of DAR(1)/D/s Queue with Discrete Mittag-Leffler [DML(α)] as marginal is done in chapter 5. The geometric distribution is taken as the special case of Discrete Mittag-Leffler distribution. Simulation study of the sample path of the arrival process is conducted. For this queueing system the stationary distribution of the system size and the waiting time distribution of an arbitrary packet is obtained with the help of matrix analytic methods and Markov regenerative theory. The quantitative effect of the stationary distribution of system size and waiting time on the autocorrelation function as well as the parameters of the input traffic is illustrated empirically. The model is applied to a real data and it is established that the model well suits this data.

In chapter 6, a new generalized counting process with Mittag-Leffler inter-arrival time distribution is introduced. This new model is a generalization of the Poisson process. The computational intractability is overcome by deriving the Mittag-Leffler count model using polynomial expansion. The hazard function of this new model is a decreasing function of time, so that the distribution displays negative duration dependence. The model is applied to a data on interarrival times of customers in a bank counter. This new count model can be simulated by Markov Chain Monte-Carlo (MCMC) methods, using Metropolis- Hastings algorithm. Our new model has many nice features such as its closed form nature, computational simplicity, ability to nest Poisson, existence of moments and autocorrelation and can be used for both equi-dispersed and over-dispersed data. The model is applied to a real data and it is established that the model well suits this data.

A new generalized counting process with Gumbel inter-arrival time distribution for the analysis of climate variability and incidence of epidemics is introduced in chapter 7. The decreasing hazard function of the new model leads to overdispersion, whereas increasing hazard function leads to underdispersion. Thus this new Gumbel count model can model over as well as under and equidispersed climate data. Hence the model can be applied to data sets on climate change where the assumption of equidispersion is violated. As climate change threatens to cause global warming, resulting rises in sea levels and temperature may influence the temporal fluctuations of epidemics like cholera, chikungunya etc., potentially increasing the frequency and duration of outbreaks of these diseases. The relationship between the number of chikungunya cases and climate change during 2005-2007 in Kerala the southern state of India is analyzed. The use of the model is illustrated with respect to two applications namely, an underdispersed data on the monthly rainfall of Kerala, southern part of India which crosses the extreme level during 2005-2008, and an overdispersed data on the daily maximum temperature in Kerala during 2005-2008.

The results of this thesis have been presented in various National and International conferences and have been submitted/accepted for publication in reputed National/International journals, the list of which is given below.

Details of Publication of papers:

[1] Kanichukattu Korakutty Jose & Bindu Abraham, "Analysis of DAR(1)/D/s queue with Quasi negative Binomial -II distribution as marginal", *Applied Mathematics*, Vol.2, No.9, 1159-1169, (2011).

[2] Jose K.K. & Bindu Abraham, "A count model based on Mittag Leffler inter arrival Times", *Statistica*, anno LXXI, n.4, 501-514, (2011).

[3] Jose K.K. & Bindu Abraham, "The analysis of discrete time queueing system with discrete Autoregressive arrivals- a review", *Research and Reviews, Journal of Statistics* (in press).

[4] Jose K.K. & Bindu Abraham, "A Counting Process with Gumbel Inter-arrival times with applications in modelling climate data", *Journal of Environmental Statistics* (in press).

[5] Bindu Abraham & Jose K.K, "Queueing models for ATM network subjected to the effects of viruses", *Proceedings of the International Workshop on Cyber Security(IWCS 2k11)*, December 6-10, 2011 (in press).

[6] Jose K.K. & Bindu Abraham, "Modeling climate change and epidemic incidence by using Gumbel count process", *Proceeding of the XI Biennial Conference of the International Biometric Society (Indian Region) on "Computational Statistics and Bio-Sciences"*, March 8-9, 2012 (in press).

[7] Bindu Abraham & Jose K.K, "Analysis of Multiserver queues with negative customers", *Proceedings of the National seminar on "Discrete Mathematics & Computational Statistics"*, 54-66, (2011).

[8] Jose K.K. & Bindu Abraham, "Analysis of DAR(1)/D/s queue with Discrete Skew Laplace as marginal distribution", *Annals of Operations Research* (under-review).

[9] Jose K.K. & Bindu Abraham, "A new count model for the analysis of climate variability and epidemic forecasting", *Journal of Transactions of the Royal Society of Tropical Medicine and Hygiene* (under-review).

[10] Jose K.K. & Bindu Abraham, "Analysis of Discrete Stable (λ, γ)/D/s queue with input traffic as DAR(1)", *Journal of Statistical Theory and Applications* (submitted).

[11] Jose K.K. & Bindu Abraham, "A new DAR(1)/D/s queue for analyzing computer virus effects", *The Journal of Mathematics and Computer Science (JMCS)* (submitted).