Dynamical behaviour of some models of interacting populations

SYNOPSIS

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The dynamical understanding between species has long been and will continue to be at the heart of many significant ecological and biological processes due to its universal existence and significance in nature. In 1952, Turing put forth a model for spatial pattern formulation of chemicals reacting and diffusing throughout tissue. Such spatial patterns in the chemicals are thought to play a role in the determination of the type of melanin is produced by the melanocytes and thus impact the formation of patterns in the mammalian coats. Starting with Turing's paper [1], diffusion has been observed as causes of the spontaneous emergence of patterns, as related to the occurrence of what he called a diffusion-driven instability. Turing also argued that the creation of such pattern could play a foremost role for spatial pattern in chemical concentrations. Over the years, Turing's idea has attracted the attention of a great number of investigators and has been successfully developed on the theoretical backgrounds. The equivalent model is a system of partial differential equations known as the reaction-diffusion model.

Segel and Jackson [2] first used reaction-diffusion system to explain pattern formation in ecological context and proposed diffusion-driven instability as an explanation for the spatial heterogeneity that is sometimes observed in predator-prey interactions. Since then, Turing instability has become an important mechanism for the emergence of interesting patterns in many model systems. Numerous papers have been published in last several decades on spatiotemporal patterns produced by reaction-diffusion systems [3, 4, 5, 6]. The majority of these works on Turing instability were focused to study the pattern formation arising from Turing instability. Some researchers have shown their interest in the study of the types of Turing patterns depending upon the choice of parameter values within Turing domain [7].

Research in last few years has shown that interacting population models exhibit a set of rich dynamical behaviours. These behaviours of interacting population models are stability around the equilibria, system persistency, local and global bifurcation analysis, chaotic behaviour, multiple attractors etc. [8, 9]. The analysis of an interacting model can be carried out by finding out its bifurcation diagram, which represents very compactly all possible modes of behaviour of the system and transition between them under parameter variations. The bifurcation diagram of an interacting population models can be very complicated, but such diagrams in many different applications may look similar i.e. topologically equivalent. The concept of topological equivalence leads to the definition of topological normal forms, i.e. polynomial forms that provide universal bifurcation diagrams and constitute one of the basic notions in bifurcation theory. Another central concept is the structural stability of a system, i.e. the topological equivalence between such a system and any other system obtained by slightly changing some parameters. The process of selection of different biologically feasible values of parameters shows that the system may undergo in a steady-state situation or may appear chaotic attractor or may follow through a sequence of bifurcations including Hopf bifurcation, saddle-node bifurcation, transcritical bifurcation, pitchfork bifurcation, homoclinic and heteroclinic orbits, Bogdanov-Takens bifurcation etc. On the domain of attraction, existence of heteroclinic orbit has important consequences.

The aim of our study is to set up and investigate some nonlinear ecological models with various functional responses which generate dynamical complexity / spatiotemporal complexity both theoretically as well as computationally. Our main goal is two-fold. On the one hand we try to identify threshold values for the parameters which correspond to bifurcations leading to a new kind of system behaviour. On the other hand our study shall contribute to the understanding of the underlying mechanisms of possible instabilities in ecosystems.

References

1. Turing, A. M., 1952, The chemical basis of morphogenesis, Philos. Trans. Roy. Soc. Lond. Ser. B, 237, 37-72.

2. Segel, L. A., J.L. Jackson, J. L., 1972, Dissipative structure: an explanation and an ecological example, J. Theor. Biol., 37, 545-559.

3. Sherratt, J. A., Eagan, B. T., 1997, Mark A. Lewis, Oscillations and chaos behind Predator-prey invasion: Mathematical artifact or ecological reality? Philos. Trans. Roy. Soc. Lond. Ser. B, 352, 21-38.

4. Jeschke, J., Kopp, M., Tollrian, R., 2002, Predator functional responses: Discriminating between handling and digesting prey, Ecol. Monogr., 72, 95-112.

5. Mukhopadhyay, B., Bhattacharyya, R., 2006, Modeling the Role of Diffusion Coefficients on Turing Instability in a Reaction-diffusion Prey-predator System, Bull. Math. Biol., 68, 293-313.

6. Sun, G. Q., Jin, Z., Zhao, Y. G., Liu, Q. X., Li, L., 2009, Spatial pattern in a predator-prey system with both self- and cross-diffusion, Int. J. Modern Phys. C, 20, 71-84.

7. Barrio, R.A., Varea, C., Aragon, J. L., Maini, P. K., 1999, A two-dimensional numerical study of spatial pattern formation in interacting Turing systems, Bull. Math. Biol., 61, 483-505.

8. Murray, J. D., Mathematical Biology (Volumes I, II), Springer-Verlag, Berlin, 1993.

9. Perko, L., Differential equations and dynamical systems, Springer (Third Edition), 2001.