SYNOPSIS

Software reliability has become an important area of research since software is an essential part of many industrial and commercial system. Even though there is no complete, scientific and quantitative measure to assess software engineering, software reliability measure is a tool to evaluate software reliability engineering. An important phase of software life cycle is the testing phase because a great deal of efforts are put down to this phase and majority of the cost is associated with this phase. One of the most challenging problems faced in the software industry is to develop reliable software and software reliability is the most significant component of continuous application availability. Software reliability is the probability that the software will be functioning without failure during a specified period of time under the given environmental conditions. Even for the two identical copies of the same software, the reliability may be different under different environments. The reliability may also vary from one machine to another, from one time point to another and also vary from one software to another. The following are the differences identified when considering software and hardware systems. (1) Software has no ageing property (2) Once a fault is removed from the software, there is no chance of occurring the failure again (3) Execution of two copies from a software program give exactly the same result. Software reliability models play an important role in developing software systems and enhancing the performance of computer software. In general, software reliability models can be classified into two types, depending on the operating domain. The most popular category of models depends on failure time, which uses the concepts such as mean time between failures and failure intensity function. The second category of software model measures reliability as the ratio of successful runs to the total number of runs. The intensity (or failure rate) function plays a pivotal role for modelling software failure time data.

The models described above are based on distribution function of failure time and reliability measures derived from it. An alternative and equivalent approach for modelling statistical data is to use quantile function. Even though both the functions convey the same information about the distribution, the methodologies and concepts based on distribution function are more popular in practice. One of the reasons for the popularity of the distribution function approach is the inferential procedure due to maximum likelihood estimation. But for the inference purposes, quantile based estimates are more robust under censoring which is very common in reliability theory. There are many simple quantile functions for empirical model building where distribution functions are not effective. In such situations conventional methods of analysis using distribution functions are not appropriate. Random numbers from any distribution can be generated using appropriate quantile functions. The characteristics derived from quantile function are more applicable in modelling and data analysis. One reason for this is that quantile based studies were carried out mostly when the traditional approach fails to give results of desired quality.In most of the cases of modelling and analysis, there has been no systematic and parallel development for replacing distribution functions by quantile functions. For the quantile function of order statistics, there are explicit general distribution forms.

Even before the nineteenth century, researchers have used the quantile based measures in various applications of statistics. The Belgian scientist Quetelet(1846) initiated the use of inter-quartile range as a quantile based measure for statistical analysis. After that researchers have focused on different applications based on quantiles such as representation of distribution by quantile functions, estimation of parameters, use of different measures etc.,. Galton (1883,1889) and Hastings et al.(1947) have introduced a family of distributions by a quantile function, which lead to the development of many quantile based families of distributions in the later period. Parzen (1979) emphasized the representation of a distribution in terms of quantile function and its role in data analysis and modelling. These were followed by Parzen(1991,1997,2004) in different areas. Gilchrist (2000) systematically presented various properties of quantile function and its use in statistical modelling.

Researchers like Parzen (1979) and Gilchrist (2000) pointed out some distinct properties and characteristics of quantile function that are useful in reliability analysis. Recently Nair & Sankaran (2009) introduced the basic concepts in reliability theory in terms of quantile functions. In reliability a single long term survivor can have a marked effect on mean life, especially in the case of heavy tailed models which are very common. In such cases quantile based estimates are generally found to be more precise and robust against outliers. In life testing experiments one need not conduct study until the failure of all the items , but only a percentage of them. In such contexts quantile based approach provides efficient estimates for survival function. This calls for a quantile based approach in reliability analysis. For more properties and applications of quantile functions in reliability analysis one could refer to Nair *et al.* (2008), Nair & Vineshkumar (2010), Nair & Vineshkumar (2011), Midhu *et al.* (2014), Midhu *et al.* (2013) and Nair *et al.* (2013).

The study of the reliability properties associated along with probability distribution function $F(x)$,density function $f(x)$ and survival function $\bar{F}(x) = 1 - F(x)$ along with various other characteristics such as failure rate, mean, percentiles, higher moments of residual life, etc,. are used for understanding how the failure time data arises in practice. A systematic study on the application of quantile function in reliability studies have been carried out by Nair & Sankaran (2009). They have discussed commonly used reliability measures in terms of quantile functions and various relationships connecting them were derived. They have also analysed quantile function model discussed in Hankin & Lee (2006) in the context of reliability analysis. Our present work extends these ideas to develop necessary theoretical framework for modelling and analysis of software reliability data based on quantile functions. This new approach provides us an alternative methodology and new models that have desirable properties. In this study we consider quantile based reliability analysis such as identifying quantile functions that are useful in software reliability, defining new families of quantile functions using various properties of reliability functions and related measures. Our present work extends these ideas to develop necessary theoretical framework for modelling and data analysis of software reliability data based on quantile functions.

After this introductory chapter the rest of this thesis is organized into five chapters. In Chapter 2 we give brief description of definitions and properties of quantile functions, some measures like hazard function, mean residual function, moments, percentiles etc., based on distribution function approach as well as quantile function approach. We explain L-moments which are alternative to conventional moments and have several advantages over usual moments. Different reliability characteristics based on distribution function and their equivalent quantile based functions are also discussed.

In Chapter 3 we introduce a software reliability model using quantile function and study its various properties. The proposed class has several desirable properties and several existing well known distributions are members of the class of distributions as special cases or through approximations. Various reliability characteristics are discussed. The parameters of the model are estimated using L-moments and the model is applied to a real data set. The approximation to two well known distributions, ie, inverse Gaussian distribution and Weibull distribution, are also carried out.

There are several reliability measures in literature to describe the patterns of failure of systems. One of the popular measures is hazard rate function. Many of the lifetime models based on quantile functions do not have explicit expression for hazard rate function so that those models can not employed for the analysis of lifetime data. Hazard quantile function which is equated to hazard rate function is used in such situation. We also present a class of distributions based on the hazard quantile function and study its properties in Chapter 4. Several existing well known lifetime distributions are members of the class of distributions as special cases or through approximations. Various reliability characteristics are discussed. The parameters of the model are estimated using the method of L-moments and the model is applied to a real data set. Non parametric estimator of $H(u)$ given in Sankaran & Nair (2009) can be employed in practice to identify the approximate lifetime model for a given dataset.

We discuss a family of distributions having inverse linear form for mean residual function $M(u)$ and study its properties in Chapter 5. The proposed class has several desirable properties and various reliability characteristics are discussed. Several existing well known distributions are members of the class of distributions as special cases or through approximations. We also derive useful characterizations connecting identities among mean residual quantile function M(u), hazard quantile function H(u), the variance residual quantile function V (u) and quantile based total time on test transform (TTT) T(u). The parameters of the model are estimated using L-moments and the model is applied to two real data sets.

Finally, we summarize major conclusions of the present work in Chapter 6. We also discuss future work that originates from the present work to be carried out in this area.