

APPLICATION OF INTUITIONISTIC FUZZY SET IN RING

Synopsis

K.Meena

Research Scholar

Department of Mathematics

Bharata Mata College, Thrikkakara, Kerala

Introduction

Fuzzy set theory exploits human knowledge in various domains where little objective and precise data are available. The objects encountered in the real physical world do not always have precisely defined criteria of membership. Inspired by human's remarkable capability to perform a wide variety of physical and mental tasks without any measurements and computations and dissatisfied with classical logic as a tool for modeling human reasoning in an imprecise environment, Lotfi A. Zadeh[42] developed the theory of fuzzy set and fuzzy logic in 1965. Fuzzy set theory is based on the idea that many non-mathematical properties cannot be adequately described in terms of crisp sets comprising those elements that fulfill a given property. The notion of fuzzy set stems from this observation to generalize the classical notion of a set and a proposition to accommodate fuzziness. The concept of fuzzy relation was also introduced by Zadeh in [43].

Further in 1967 J.A.Goguen [21] introduced the concept of L-fuzzy sets, a generalization of fuzzy sets, where the membership function takes values in a lattice L . The set of all L-fuzzy sets on a set U can be equipped with whatever operations L has, and these inherited operations obey any law valid in L which extends point wise.

Intuitionistic fuzzy set introduced by K.T. Atanassov [8] in 1983 are extensions of the standard fuzzy sets which allows the assessment of the elements by the membership and non-membership function denoted by μ and γ which belong to the real unit interval $[0,1]$. Intuitionistic fuzzy set allows one to address the positive and negative side of an imprecise concept separately and does not insist that these assessments be exactly complementary such that, for $x \in X$, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$, where A is a subset of the universal set X . The number $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$ quantifies the degree of indeterminacy associated with x and A .

Intuitionistic L-fuzzy set introduced by K.T. Atanassov and Stoeva [9] is a generalization of L-fuzzy set where the membership and non-membership function takes values in a bounded lattice with maximal element 1 and minimal element 0, respectively.

In 1975 Zadeh [44] made an extension of the concept of fuzzy set by an interval-valued fuzzy set. Later in 1989, K.T. Atanassov and G. Gargov [10] proposed interval-valued intuitionistic fuzzy set based on the comparative analysis of interval-valued fuzzy sets and intuitionistic fuzzy sets. Intuitively, the extension of intuitionistic fuzzy sets to interval-valued intuitionistic fuzzy sets furnishes additional capability to handle vague information.

In 1971 A. Rosenfeld [41] introduced the concept of a fuzzy subgroup of a group. The concept of fuzzy subrings and fuzzy ideals were introduced and redefined in [12, 33, 34]. Later N. Ajmal and K.V. Thomas [2 to 6] studied the lattice structure of fuzzy algebraic structures and also proved its modularity. Based on that many concepts in Algebra were extended to the fuzzy setting by various mathematicians [1, 17 to 20, 24, 30, 31, 35, 36]. The developments of Algebra in fuzzy setting are very much evident in the books of D.S. Malik and J.N. Mordeson [37, 38] who initiated the study of fuzzy algebraic structures on a complete Heyting algebra. During the development of fuzzy Algebra, mathematicians were on the process of extending the concepts of Algebra in the intuitionistic fuzzy setting.

Intuitionistic fuzzy set approaches to various universal algebraic concepts started with R.Biswas's [13] intuitionistic fuzzy subgroups in 1989. In 2000 Kumar De and others [29] discussed completely the operations on intuitionistic fuzzy sets. The concepts of group theory constituted a new building block of the theory of intuitionistic fuzzy groups. In 2003 B.Banerjee and D.K.Basnet [11] introduced intuitionistic fuzzy subrings and ideals and studied some properties of intuitionistic fuzzy ideals under ring homomorphism. In 2006 B.Davvaz and W.Dudek [16] introduced and investigated intuitionistic fuzzy H_v ideal of a H_v ring. K.H.Kim and others [23] applied the notion of Intuitionistic fuzzy sets to semigroups and studied the notions of interior ideals, bi-ideals etc.

In 2011 M.F.Marshdeh and A.R.Salleh[39] formulated intuitionistic fuzzy rings on the notion of intuitionistic fuzzy space. Also they discussed the relation between intuitionistic fuzzy rings with ordinary rings and fuzzy rings. Though many researchers have applied the notion of intuitionistic fuzzy sets to ring theory, a systematic study of intuitionistic L-fuzzy sets in ring theory was carried out by K.Meena and K.V.Thomas [i to v].

Relations play a prominent role in the real physical life. In conventional Mathematics there are many concrete relations which enhanced its growth. Fuzzy relations are instrumental in modeling ambiguity. But intuitionistic fuzzy relations incorporate vagueness to an additional degree. In 1995 H.Bustince and P.Burillo [14] introduced the concept of intuitionistic fuzzy relations on a set and studied their properties. They also studied the composition of intuitionistic fuzzy relations, and have analyzed the different properties of this composition according to the choice of t-norms and t-conorms. Moreover they studied the effect of Atanassov's operators on the properties of the intuitionistic fuzzy relations. In particular they established the conditions that an intuitionistic fuzzy relation must fulfill in order that the properties of reflexivity, symmetry and transitivity are maintained by means of operators. Since 2001 to 2006 Kul Hur and his colleagues [25-28] investigated intuitionistic fuzzy equivalence relations and intuitionistic fuzzy congruences on a groupoid. Moreover they studied the lattice structure of intuitionistic fuzzy congruence relation and its modularity. Also they discussed the isomorphism between the lattice of intuitionistic fuzzy

congruence relation and the lattice of intuitionistic fuzzy normal subgroup in a groupoid.

In 1995 H.Bustince and P.Burillo [15] discussed the way of constructing interval-valued intuitionistic fuzzy sets from intuitionistic fuzzy sets. In 2001 T.K.Mondal and S.K.Samanta[40] defined the topology of interval-valued intuitionistic fuzzy set and studied its properties. They showed that the category of topological spaces of interval-valued intuitionistic fuzzy sets and continuous functions form a topological category. Later M.Akram and W.Dudek [7] introduced interval-valued intuitionistic fuzzy Lie ideals of Lie algebras and discussed certain properties. In 2009 H.Hedayati [22] studied interval-valued intuitionistic (S, T) fuzzy substructures in semirings with respect to t-norm T and s-norm S and the characteristic properties are described. In particular, new interval-valued intuitionistic (S, T) -fuzzy subsemirings (ideals) are constructed. In 2012 P.K.Lim and others [32] investigated the lattice structure of various sublattices of the lattice of interval-valued fuzzy subrings of a ring. Moreover they introduced a special class of interval-valued fuzzy ideals and proved that it forms a modular sublattice of the lattice of interval -valued fuzzy ideals of a ring.

In the context of the evolution of fuzzy set theory, we pursued research in the area of intuitionistic fuzzy set theory. In the thesis we discussed and studied completely the properties of intuitionistic L- fuzzy subrings and ideals under various operations defined by us. The concept of image and inverse image of an intuitionistic L-fuzzy set under ring homomorphism and its characteristics are studied. In this direction some classical results of ring theory are established by the application of intuitionistic L-fuzzy subrings. Further we studied the lattice structure of intuitionistic L-fuzzy equivalence relations and that of intuitionistic L-fuzzy congruence relations along with its modularity. We established an isomorphism between the lattice of intuitionistic L-fuzzy ideals and that of intuitionistic L-fuzzy congruence relations of a ring. In the wake of development of interval-valued intuitionistic fuzzy set, here we defined interval-valued intuitionistic fuzzy subring and interval-valued intuitionistic fuzzy ideal of a ring. The relationship between the level subset of an interval-valued intuitionistic fuzzy set and interval-valued intuitionistic fuzzy subring (interval-valued intuitionistic

fuzzy ideal) are studied. It is proved that a subclass of interval-valued intuitionistic fuzzy ideal form a complete modular sublattice of the lattice of interval-valued intuitionistic fuzzy ideals of a ring.

Chapter wise Summary

The entire thesis is divided into seven chapters and each chapter is further subdivided into a number of sections.

In chapter 1, a brief history and the subsequent advancement in the theory of fuzzy sets, L-fuzzy sets, intuitionistic fuzzy sets, intuitionistic L-fuzzy sets and interval-valued intuitionistic fuzzy sets are furnished. Also the research work carried through in the thesis is presented in a concise manner.

In Chapter 2, some basic definitions, results and properties of rings and lattices, L-fuzzy sets, intuitionistic L-fuzzy sets and interval-valued intuitionistic fuzzy sets which are mandatory for the research work is accentuated.

The concepts of intuitionistic L-fuzzy subrings (ILFSR) and intuitionistic L-fuzzy ideals (ILFI) of a commutative ring R with identity are introduced and discussed in chapter 3. Firstly, we defined certain operations on Intuitionistic L-fuzzy sets (ILFS). The properties of ILFSR and ILFI under these operations are studied. The notion of Intuitionistic L-fuzzy ideal generated by an intuitionistic L-fuzzy set is introduced and some of its characterizations are provided. We proved that the generator of the union of two ILFI is its sum. Subsequently the residual quotient of an intuitionistic L-fuzzy set is introduced and proved that it is an intuitionistic L-fuzzy ideal. Lastly, we introduced intuitionistic L-fuzzy coset and studied the quotient ring of a ring by the membership and non-membership functions of an ILFI.

The image and inverse image of ILFS under ring homomorphism and its properties are provided in chapter 4. We constructed an intuitionistic L-fuzzy set of a quotient ring and proved that it forms an intuitionistic L-fuzzy ideal. The correspondence theorem for intuitionistic L-fuzzy ideal under ring epimorphism is

established. Also we defined the intuitionistic L-fuzzy quotient subring (ILFQSR) of an ILFSR relative to an ideal of a ring. An ILFI of an ILFSR is defined and certain results are proved. Besides, the support of an intuitionistic L-fuzzy ideal is defined and its characterization is discussed. Moreover we proved that under a ring epimorphism the image and inverse image of an ILFI of an ILFSR forms an ILFI of the image and inverse image of the ILFSR. The homomorphism and isomorphism of an intuitionistic L-fuzzy subring is discussed. As in the case of ring theory, the fundamental theorem of ring homomorphism and the third isomorphism theorem are established in the intuitionistic L-fuzzy setting.

Chapter 5 emphasizes the study of intuitionistic L-fuzzy equivalence relation (ILFER) and Intuitionistic L-fuzzy congruence relation (ILFCR) on a ring along with their lattice structures. Firstly, we introduced intuitionistic L-fuzzy relation (ILFR). Consequently some basic concepts of ILFR and ILFER under certain operations are discussed. In a sequel, an ILFER generated by an ILFR and the intuitionistic L-fuzzy transitive closure of an ILFR is defined. Using these operations the join of two ILFER is constructed. Further it is proved that ILFER forms a complete lattice.

In the next section we defined ILFCR and studied some of their properties under the operation given in the previous section. Besides the lattice structure of intuitionistic L-fuzzy congruence relation is provided. Moreover it is established that the class of ILFCR forms a complete modular lattice.

The lattice structure of intuitionistic L-fuzzy ideals of a ring is initially discussed in chapter 6. This is followed by an introduction to a subclass of intuitionistic L-fuzzy ideal having the same grade at the identity element of a ring and we later proved that this subclass forms a complete modular sublattice of the lattice of intuitionistic L-fuzzy ideal. We established the relationship between intuitionistic L-fuzzy ideal and intuitionistic L-fuzzy congruence of a ring. Further we proved the existence of an isomorphism between the lattices of intuitionistic L-fuzzy ideal and that of intuitionistic L-fuzzy congruence relation on a ring.

Eventually, we carried out a study of interval-valued intuitionistic fuzzy set (IVIFS) of a ring along with its level subsets in chapter 7. We investigated the lattice

structure of interval-valued intuitionistic fuzzy subrings and that of various sublattices of the lattice of interval-valued intuitionistic fuzzy ideals of a ring.

Firstly, we defined and illustrated interval-valued intuitionistic fuzzy subring (IVIFSR) and interval-valued intuitionistic fuzzy ideal (IVIFI) of a ring. Some properties of IVIFSR and IVIFI are studied. The supremum property (sup property) of an interval-valued intuitionistic fuzzy set is introduced. The lattice structure of IVIFSR and that of IVIFI is provided. This leads us to the formation of various lattices and sublattice of interval-valued intuitionistic fuzzy ideals. As in the case of fuzzy setting, in intuitionistic fuzzy setting also the notion of level subset is found to be an important tool for establishing various ring theoretic properties. Here we characterized an IVIFSR and IVIFI by way of its level subset. Some properties of the level subsets of IVIFI are established. Also we provided certain characterizations of IVIFSR and IVIFI under ring epimorphism. A subclass of interval-valued intuitionistic fuzzy ideal of a ring with the same tip is introduced and proved that it forms a sublattice of the lattice of interval-valued intuitionistic fuzzy ideal.

Finally we introduced an interesting subclass of the class of interval-valued intuitionistic fuzzy ideal with the same tip and proved that it forms a modular sublattice of the lattice of interval-valued intuitionistic fuzzy ideal of a ring.

The subject matter of the thesis depends on the following papers:

- i. K. Meena and K. V. Thomas, Intuitionistic L-fuzzy Subrings, International Mathematical Forum, 6 (2011), No.52, 2561–2572.
- ii. K. Meena and K. V. Thomas, Intuitionistic L-Fuzzy Rings, Global Journal of Science Frontier Research (F) Vol 12, 2012, No. 14-F, Art.2, 17- 31.
- iii. K.Meena and K.V.Thomas, Intuitionistic L-Fuzzy Congruence on a Ring, Notes on Intuitionistic Fuzzy Sets, Vol.19, 2013, No.1, 7-20.
- iv. K.Meena and K.V.Thomas, Interval-Valued Intuitionistic Fuzzy Ideals of a Ring (Accepted in the journal -Advances in Fuzzy Sets and Systems).
- v. K.Meena and K.V.Thomas, Intuitionistic L-Fuzzy Ideals and Congruences of a Ring (Communicated).