SOME PROBLEMS IN INTUITIONISTIC FUZZY OPTIMIZATION AND DECISION MAKING

Synopsis

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1. Introduction:

The desire for optimality is inherent in humans. Various optimization techniques and methods have already been successfully applied to solve problems with a well defined structure. Such optimization problems are usually well formulated by specific crisp objective functions and specific system of crisp constraints, and solved by classical mathematical tools. However, real world situations are often not deterministic or crisp in nature. There exist various types of uncertainties in social, industrial and economic systems, such as randomness of occurrence of events, imprecision and ambiguity of system data and linguistic vagueness, etc. which arise from many ways, including errors of measurement, incomplete knowledge expression, and the subjectivity and preference of human judgment, etc. These uncertainties can be categorized as uncertainty due to

randomness (aleatory uncertainty), uncertainty due to lack of specification and uncertainty due to vagueness, imprecision and ambiguity about the set boundaries (fuzziness).

Vagueness is associated with the difficulty of making sharp or precise distinctions, i.e. it deals with the situation where the information cannot be valued sharply or cannot be described clearly in linguistic term, such as preference related information. This type of fuzziness is usually represented by membership function which reflects the decision-maker's subjectivity and preference on the objects. Ambiguity is associated with the situation in which the choice between two or more alternatives is left unspecified, and the occurrence of each alternative is unknown owing to deficiency in knowledge and tools.

Fuzzy and Intuitionistic fuzzy set theoretical tools have been proved better tools to deal with models where uncertainty, ambiguity and vagueness are inherently present. The introduction of fuzzy set(FS) theory by Lotfi A. Zadeh in 1965, decision making become more acceptable than the previous one. While decision making under conditions of risk have been modeled by probabilistic decision theories and game theories, fuzzy decision theories attempt to deal

with the vagueness and non-specificity inherent in human formulation of preferences, constraints and goals.

Several classes of decision making problems are usually recognized. According to one criterion, G.J.Klir in [10] classified decision problems as those involving a single decision maker and those involving several decision makers. These problem classes are referred to as individual decision making and multiperson decision making, respectively. Similar to multiperson decision making, we have multicriteria decision problems, where relevant alternatives are evaluated according to a number of criteria.

A system with vague and ambiguous information can be better formulated and solved by fuzzy or/and intuitionistic fuzzy optimization techniques rather than by using classical mathematical optimization techniques.

Linear programming has been an important and most frequently applied optimization technique for solving real life problems. It is a technique used for determining an optimum schedule of interdependent activities based on the available resources. Linear programming deals with the optimization of a function of a variable, known as objective function, subject to a set of linear equations or inequalities.

It has been used to solve problems related to the assignment of jobs to machines, blending, product mixing, advertising media selection, least cost diet, distribution, transportation, investment portfolio selection and so on. Intuitionistic fuzzy optimization techniques are developed on the basis that one may be able to express the constraints and objective functions in an IF context.

1.1. Objective of the thesis.

The main objective of the thesis are,

(1) To enrich existing literature on Intuitionistic fuzzy set theory.

(2) To extend some basic concepts in Fuzzy set theory to Intuitionistic fuzzy set theory and to apply them in the context of optimization and decision making.

2. Literature survey

The fuzzy set theory was introduced by L. A. Zadeh in 1965. The classical results in fuzzy set theory are found in the works of L. A. Zadeh and Zimmerman [27], [28], [30] and[31]. By 1983 fuzzy set theory was extended to intuitionistic fuzzy set(IFS for short) theory by K. T. Atanassov in his work [6], [7] and [8]. IFS theory contains three functions namely membership, nonmembership and hesitancy. Later in 1989 K. T. Atanassov and Gargov developed Interval valued

intuitionistic fuzzy set (IVIFS for short) theory [5].

Ranking of IFS and IVIFS plays an important role in decision making. Almost everything that a human being does involves decisions. Therefore, to theorize about decisions is almost the same as to theorize about human activities. Multicriteria decision making methods using IFS theoretical tools were introduced in decision theory in 2007 by Z. S. Xu [21], [22], [23]. Xu introduced aggregation operators to rank IFSs. In 2008 Lakshmana Gomathinayagam introduced new ranking methods for IFS [11] and in [12]. Authors including Lakshmana Gomathinayagam rectified error in the ranking methods for IVIFS given by Xu. In this thesis we have proposed new score functions, accuracy functions and an aggregation operator which helps the decision maker to rank the alternatives in IF as well as IVIF context.

Linear programming has been an important and most frequently applied optimization technique for solving real life problems. Plamen P. Angelov in 1994 introduced methods to solve optimization problems in fuzzy context [2]. Later he extended these methods to solve optimization problems in IF context as mentioned in [3]. We have studied the methods given by Plamen P. Angelov and that

given by Nagoorgani A. [17], [18], and we have developed new algorithm for solving Assignment problem and Transportation problem.

3. Chapter wise summary of the thesis

We begin with an introduction to fuzzy sets, intuitionistic fuzzy sets (IFSs for short) and interval valued intuitionistic fuzzy sets(IVIFSs for short)

Our main reference for fuzzy set theory is [10] and for intuitionistic fuzzy set theory is [8]. But for optimization theory we refer [2], [3] and [17].

The thesis is divided into nine chapters.

Chapter 0 is an introduction, which gives an outline of the entire thesis.

Chapter 1 consists of basic definitions and preliminary results which are used in the subsequent chapters.

In chapter 2, we recall the definition of (α, β) cut of an IFS as given by P. K. Sharma [19] and we have defined strong (α, β) cut of IFS and study its properties. Also two special IFSs are defined. Using these (α, β) cuts, three decomposition theorems for IFSs are discussed and illustrated.

In chapter 3, we introduce a method of solving multi-attribute

decision making problems in intuitionistic fuzzy environment. Certain drawbacks in the definition of accuracy function put forward by Zhi Pei and Li Zheng [29] have been identified in this chapter, and we propose a new optimization model and a new accuracy function. This new accuracy function is then used to solve a revised optimization model to estimate the relative degree of importance of membership, nonmembership and hesitation for IFSs and thus to rank the alternatives. Its computational procedure is discussed and illustrated.

Chapter 4 analysis IVIFSs. We propose a new score function, which ranks all comparable and incomparable IVIFSs. Various methods for the comparison of IVIFSs is recalled in section 4.2. Comparative study of three score functions in the literature is elicited in section 4.4. Decision making with the new score function is described and illustrated. Application of this new score function in multi-person multi-criteria DM problem, where each decision maker is treated with different weights whose sum in one, is explained in section 4.6.

In chapter 5, a weighted accuracy function, theorems related to it, and a decision function which consider decision makers attitude

is explained and illustrated.

Chapter 6 focuses on (α, β) level, closure and interior of IFSs given by K. T. Atanassov. New theorems on (α, β) level are proved and illustrated in this chapter. In section 6.2, theorems on closure and interior of IFSs are defined and illustrated. Theorems related to the operators, $\boxplus A$ and $\boxtimes A$ of IFS are given in section 6.3. Some results on nested subsets and supersets of an IFS A are obtained and illustrated by suitable example in this section.

The principle of intuitionistic fuzzy assignment problem and intuitionistic fuzzy transportation problem is critically studied in Chapter 7. Applying the concept of IFS, an assignment problem is reformulated using degrees of acceptance and degrees of rejection of constraints where sum of both degrees is required to be less than or equal to one. We propose an algorithm for solving an IF assignment model. It is further generalized by treating the costs of assigning jobs to machines(cost of assigning alternatives) as interval valued intuitionistic fuzzy quantities. Section 7.3 is an illustration. In section 7.4 we propose an algorithm for solving a transportation problem in interval valued intuitionistic fuzzy context.

Chapter 8 deals with Multi-criteria decision making problems.

In this chapter, we propose an aggregation operator to aggregate the criteria for intuitionistic fuzzy alternatives, both in intuitionistic fuzzy and interval valued intuitionistic fuzzy cases. We also propose score function and accuracy function to rank the alternatives in IVIF context. We have illustrated these methods and have applied the new aggregation operator in an assignment problem also.

Chapter 9 gives the brief summary of the work.

4. Conclusion

Our thesis entitled **Some Problems in Intuitionistic Fuzzy Optimization and Decision Making** focusses on optimization and decision making in Intuitionistic Fuzzy Sets. After studying various IF set theoretical tools existing in the literature for decision making models, we have modified some of the decision making models, proposes new score functions, accuracy functions and aggregation operators which are more elegant than the existing functions. Also, we propose new algorithm for solving IF assignment problem as well as transportation problem. Apart from the topic mentioned in the title, we have discussed and proved three decomposition theorems for IFSs.

5. List of publications

[1] Decomposition theorems of an intuitionistic fuzzy set, *Note on Intuitionistic Fuzzy Sets* Vol. 18(2012) No.2, 31-36.

[2] Note on Multi attribute decision making in intuitionistic fuzzy context, *Note on Intuitionistic Fuzzy Sets,* Vol.19 (2013) N0.1, pp. 48-53.

[3] Multicriteria Decision Making Under Interval Valued Intuitionistic Fuzzy Context, *Journal of Fuzzy Mthematics* Vol. 21, No. 4(2013)

[4] Multi attribute decision making using decision makers attitude in intuitionistic fuzzy context, *International Journal of Fuzzy Mathematics and Systems* Vol.3, NO.3(2013), pp. 189-195.

[5] Aggregation of Intuitionistic Fuzzy Sets using(α, β)cuts, *Journal of current studies: Proceedings of National Conference on Modelling, Optimizattion and computational Techniques, S.N.M. College Maliankara*(2013), pp.79-84.

[6] Algorithm for solving an Assignment model in IF context, *International Journal of Fuzzy Mathematics and Systems* Vol.3

No.5(2013) pp. 345-349.

[7] Aggregation operator, Score function and Accuracy function for Multicriteria decision problems in intuitionistic fuzzy context, *Note on Intuitionistic Fuzzy Sets*, Vol. 20, No. 1, (2014)

[8] More on(α, β) level, Closure and Interior, in intuitionistic fuzzy context, *Georgian Educational Mine International Journal*,(2014), Vol. 1, No. 1, pp.73-77.

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