Introduction

If we make some dots on the piece of paper and then join some of them by line segments we obtain figures as given in fig.1.1

![Fig. 1.1](image)

Such figures are known as graphs. This terminology in universally accepted. The terminology ‘graph’ should not be confused with the traditional graph which is drawn on the graph paper by choosing proper axis.

‘Graph Theory’ is one of the important branch of Mathematics. The contribution of ‘Graph Theory’ is remarkable as many complicated mathematical structures can be built just with few basic concepts.

Formally we define ‘graph’ as follows:- A graph G is a pair ( V(G) , E (G)) where V ( G) is a nonempty finite set of elements known as vertices and E (G) is family of unordered pairs of elements of V (G) known as edges. Some of the interesting graphs under study are given in fig 1.2

![Fig.1.2](image)
When the dots or lines in a graph are labeled with numbers we call it a ‘labeled graph’. Thus, labeling of the vertices of a graph G is assignment of distinct natural numbers to vertices of G. This labeling induces a natural labeling of the edges called edge labels or edge weights. The label of an edge uv is absolute value of difference of labels of u and v . In the other type of labeling, the edges are assigned labels or numbers and this labeling of edges induces natural labeling of vertices. This type of labeling the graph is called even or odd vertex labeling. There are various types of vertex labeling. Some such labelings are complete labeling, graceful labeling, k-equitable labeling, harmonious labeling, cordial labeling, α-labeling, elegant labeling, H-labeling, graceful labeling, set labeling, magic labeling, anti-magic labeling, set magic labeling, Σ-labeling, multiplicative labeling, strongly multiplicative labeling, prime labeling and orthogonal labeling.

One of the earliest types of labeling is graceful labeling. The graphs which have graceful labeling is called graceful graph. Formally we define graceful graph as follows:-

Let G be a graph with q edges. Let f be labeling of G such that the set of labels of vertices is a subset of \{0,1,2,3,......,q\} and the set of the edge labels is from set \{1,2,3,......,q\} Then the labeling f is said to be graceful and graph G is called graceful graph. To find if a graph is graceful or not, is one of the major problems in Graph Theory.

Labeling methods trace their origin to one introduced by Rosa[1] in 1967 and one given by Graham and Sloane in 1980. Rosa called a function f of labels of vertices of graph G as β-valuations of graph G with q edges if f is an injection from vertices of G to the set \{0,1,2,3,......,q\} such that each edge uv is assigned the label \|f(u) − f(v)\| , the resulting labels are distinct. Golomb subsequently called such labelings as graceful labelings. The examples of graceful labelings is shown in fig. 1.3
In graph of fig.1.3 a the vertex are labeled from set \{0,1,2,3,4,5,6\} in such a way that the edges carry labels from set \{1,2,3,4,5,6\}. In fig.1.3 b the vertex are labeled from set \{0,1,2,3,4,5,6\} and edges carry labels from set \{1,2,3,4,5,6\}

The latest interesting labeling of edges have given rise to new type of graphs called even-vertex graceful graphs. The formal definition of even-vertex graceful graphs is as follows:- A graph is even-vertex graceful graph if there exists a injective map 

\[ f: E(G) \rightarrow \{1,2,......, 2q\} \]

so that the induced map

\[ f^*: V(G) \rightarrow \{0,2,4,.....,2(k-2)\} \]

defined by \[ f^*(x)= \sum f(xy) \pmod{2k} \]

where \( k = \max\{p,q\} \) makes all labelings distinct.

Similarly, the edge-odd graceful graph can be defined as follows:- A (p,q) connected graph is edge-odd graceful graph if there exists an injective map \( f: E(G) \rightarrow \{1,3,..........., 2q-1\} \) so that induced map,

\[ f^*: V(G) \rightarrow \{0,1,2,......,2k-1\} \]

defined by \( f^*(x)= \sum f(xy) \pmod{2k} \) where the vertex \( x \) is incident with other vertex \( y \) and \( k = \max\{p,q\} \) makes all the edges distinct and odd.

Even-vertex graceful labeling of path, circuit, star, wheel and some extension friendship graphs is proved by Solairaju and Muruganatham [29] while edge-odd gracefulfulness of graph \( S_2 \square S_n \) is proved by Solairaju, Vimala and Sasikala [30].
Some labelings are already defined for corona graph $C_n*K_1$. As these graphs are symmetrical we can find that more labelings can be applicable for corona graph $C_n*K_1$. Different labelings are defined for graphs like trees, cycles, book and wheels but corona graph $C_n*K_1$ are not much looked upon. It is a pure research wherein the knowledge obtained can form base for further research. The researchers in Graph Theory are primarily concerned with conditions and trends that are developing with different labelings of graphs.

Labelings of graphs are used in Graph Theory. Graph Theory have found applications in operational research, organic chemistry, electrical networking theory and computing. So hereby, we try to explore some more labeling for corona graphs $C_n*K_1$. 