Introduction:

The queuing analysis is to offer a reasonably satisfactory service to waiting customers. It is not an optimization technique but it determines the measure of performance of waiting lines, such as the average waiting time in queue and the productivity of the service facility, which can then be used to design the service installation. In general we do not like to wait. But reduction of the waiting time usually requires extra investments. To decide whether or not to invest, it is important to know the effect of the investment on the waiting time. So we need models and techniques to analyze such situations.

A simple Poisson tandem queue with infinite buffers has been studied in the literature, (Brockett, Weibo and Yang 1999). They have formulated and solved a number of general questions using the theory of stochastic differential equations together with a systematic use of the Itô calculus as their fundamental modeling methodology. Miyazawa, Sakume and Yamaguchi (2007) studied asymptotic behaviors of the loss probability for a finite-buffer queue. Their model includes a many-server queue and is described by a truncated quasi birth–death (QBD) process. To avoid large state space, cases that may develop in some systems such as open-networks of mixed finite- and infinite-buffer queues with phase-type distributed service and inter-arrival times, systems can be decomposed in separate systems. This is because matrix-geometric, spectral expansion and other techniques can solve such systems efficiently. However, decomposition is not possible for systems that involve task loss and feedback such as the one we will consider. Retrial queuing systems shown to be good mathematical models for processing information transmission in many telecommunication networks such as telephone switching systems, cellular mobile networks, local and metropolitan area networks under different protocols of random multiple access, etc., (Artalejo 1999), for example. It seems that majority of publications in this area is considering systems with the stationary Poisson arrivals. To catch the typical features of traffic in modern telecommunication networks such as correlation and burstiness, retrial queuing systems with the batch Markovian arrival processes
(BMAP) or its partial case, Markovian arrival processes (MAP) have been studied. Retrial queueing systems with the MAP are described by the level dependent QBD processes, see Artalejo, (1999) and Diamond and Alfa (1998), for example. Klimenok and Taramin (2007) studied a tandem queue with loss and blocking. The arrival is assumed to be BMAP. They derived the condition for stability and calculated performance measures. Flow-based networking can help addressing current performance issues in convergent internet protocol networks. They proposed a model that allows the blocking probability calculation in the case where flows have different rates with known mean and SD and verified their model by simulation. Feedback queues play an important role in real-life service systems, where tasks may require repeated services. Tandem queues with feedback have been widely studied in the literature. Applications of such systems are in manufacturing systems and computer networks. Tang and Zhao (2008), assumed Markovian arrival and service times. However, upon completion of a service at the second station, the task either leaves the system with probability $p$, or goes back, along with all tasks currently waiting in the second queue, to the first station with probability $1 - p$. They developed a method to study properties of exactly geometric tail asymptotic as the number of tasks in the other queue increases to infinity. Their objective was to illustrate how to deal with a block generating function of $GI/M/1$ type.

It should be noted that none of the tandem models, including Jackson’s (1957) and Brockett, Weibo and Yang (1999), have considered splitting feature with analytical solutions. Although the terminology has been used in a different context. This research, will partially fill this gap and this is one of the novelties of the model.