2.0 **LITERATURE REVIEW**

**Banach (1922),** in this paper, author has conceptualize that let \( K \) be a complete metric space in which the distance between two points \( P \) and \( Q \) is denoted \( d(P,Q) \). And let \( F: K \rightarrow K \) be a contraction, i.e. there exists \( c \in (0, 1) \) such that for all \( P, Q \in K \), then \( d(F(P), F(Q)) \leq c \ d(P,Q) \). Then \( F \) has a unique fixed point, i.e. there exists a unique \( A \in K \) such that \( F(A) = A \). Author has also given the following definitions:

(i) A distance on a set \( K \) is a function \( d: K \times K \rightarrow \mathbb{R} \) satisfying

1. For all \( P, Q \in K \), \( d(P,Q) \geq 0 \);
2. \( d(P,Q) = 0 \) if and only if \( P = Q \);
3. For all \( P,Q,R \in K \), \( d(P,Q) \leq d(P,R) + d(R,Q) \); (triangular inequality.)

(ii) A sequence \( \{P_n\} \) of elements of a metric space \( K \) is a Cauchy sequence if for all \( Q > 0 \), there exists \( N \in \mathbb{N} \) such that for all \( n, m > N \), then \( d(P_n, P_m) < Q \). A sequence \( \{P_n\} \) of elements of a metric space \( K \) converges to a limit \( A \in K \) if for all \( Q > 0 \), there exists \( N \in \mathbb{N} \) such that for all \( n > N \), then \( d(P_n, A) < Q \).

(iii) A metric space \( K \) is a complete metric space if any Cauchy sequence \( \{P_n\} \) of elements of \( K \) converges to an element \( A \) of \( K \).

**Gahler (1964),** in this paper, author has invented a new metric space namely 2-metric space. He proved that the Banach Fixed point theorem also applicable in 2- metric space.

**Zadeh (1965),** in this paper, author has introduced the concept of fuzzy.

**Tramosil and Michalet (1975),** in this paper, authors have introduced the concept of fuzzy metric space, which opened an avenue for further development of analysis in such spaces.
Sesha (1982), in this paper, author has introduced commutating and weakly Commutating mapping and establish some fixed point results take f and g as Commutating and weakly Commutating mappings.

Dhage (1984), in this paper, author has invented another two metric spaces, D-Metric spaces and Cone Metric spaces respectively. They proved that the Banach fixed point theorem also satisfy the D-Metric and cone metric spaces.

Khan et al., (1984), in this paper authors have proved that let \((X, d)\) be a complete metric space, \(\psi\) be an altering distance function, \(c \in [0, 1)\) and \(T : X \rightarrow X\) satisfying \(\psi(d(Tx,Ty)) \leq c\psi(d(x, y))\), for all \(x, y \in X\). Then \(T\) has an unique fixed point.

Jungck (1986) introduced the Compatible mapping and prove some new results on fixed point theorem. On the other hand Fisher(1978) prove fixed point theorem taking as an increasing function from \(\mathbb{R}^+\) to \(\mathbb{R}^+\)and more than one metric space. After that many mathematician work in this line take different type of mapping such as Non-expansive mappings, self mapping, multivalued mapping, sequence of mappings, operator in Hilbert spaces and another mappings in metric spaces, Hilbert spaces, Banach spaces. Recently fixed point theorem or Contraction mapping is applied to find out the existence and uniqueness solution of higher order differential and integral equations.

Pant (1994), in this paper, Author has obtained some common fixed point results on fuzzy metric spaces generalizing the earlier results (Vasuti,1999; Som, 1985).

Subrahmanyam (1995), in this paper, author has consequently verified some more metric fixed point results more generalized to fuzzy metric spaces in due course of time and introduced by various authors (Grabiec,1988; Vasuti,1999).
**Alber and Guerre-Delabriere (1997)**, in this paper, authors have defined that Let $(E, \| \cdot \|)$ be a Banach space and $C \subseteq E$ a closed convex set. A map $T : C \to C$ is called weakly contractive if there exists an altering distance function $\psi : [0, \infty) \to [0, \infty)$ with $\lim_{t \to \infty} \psi(t) = \infty$ such that $\|Tx - Ty\| \leq \|x - y\| - \psi(\|x - y\|)$, for all $x, y \in X$. Further they have also proposed a theorem that let $H$ be a Hilbert space and $C \subseteq H$ a closed convex set. If $T : C \to C$ is a weakly contractive map, then it has a unique fixed point $x^* \in C$.

**Jungck and Rhoades (1998)**, in this paper, author has proposed and fixed point theorems for set-valued functions without appeal to continuity. This is done via the concept of compatible (weak) maps and generalized Meir – Keeler contractions called $(\varepsilon, \gamma)$ contractions.

**Pant (1998)**, Two common fixed point theorems have been proved by using minimal type commutativity and contractive conditions. The last theorem extends known results on compatible maps to a wider class of mapping.

**Huiqin (2001)**, in this paper, author has use the monotone iterative technique and a comparison result to prove some existence theorems of minimal and maximal solutions of periodic boundary value problems for nonlinear first order impulsive functional differential equations in Banach spaces then he give some applications to boundary value problems for second order functional differential equations in Banach spaces.

**Rhoades (2001)**, in this paper, author has proposed and verified a theorem that let $(X, d)$ be a complete metric space, $\psi$ be an altering distance function and $T : X \to X$ satisfying $d(Tx, Ty) \leq d(x, y) - \psi(d(x, y))$ for all $x, y \in X$. Then $T$ has a unique fixed point.
Sharma and Deshpande (2002), in this paper, authors have proved a convergence theorem of a generalised Ishikawa iteration sequence for two multi-valued strongly pseudo-contractive mappings by using an approximation method in real uniformly smooth Banach spaces. He generalize and extend the results of Chang and Chang, Cho, Lee, Jung and Kang.

Mustafa and Sims (2003, 2006), in this paper, authors have introduced a new concept of generalized metric spaces, called $G$-metric spaces. In such spaces every triplet of elements is assigned to a non-negative real number. Based on the notion of $G$-metric spaces,

Ran and Reurings (2004), in this paper, authors have proposed that let $(X, 3)$ be a partially ordered set such that every pair $x, y \in X$ has a lower and an upper bound. Let $d$ be a metric on $X$ such that the metric space $(X, d)$ is complete. Let $f : X \to X$ be a continuous and monotone (i.e., either decreasing or increasing with respect to 3) operator. Suppose that the following two assertions hold: (1) there exists $k \in [0, 1)$ such that $d( fx, fy ) \leq kd(x, y)$ for each $x, y \in X$ with $x \leq y$; (2) There exists $x_0 \in X$ such that $x_0 \in fx_0$ or $x_0 \in fx_0$. Then $f$ has an unique fixed point $x^* \in X$.

Xu (2004), in this paper, author has investigated that “every cone metric space is a topological space”. He has also generalized the concepts diametrical contractivity and asymptotical diametrical contractivity to cone metric spaces.

Nieto and Lopez (2005), in this paper, authors have proposed that let $(X, a)$ be a partially ordered set and suppose that there exists a metric $d$ in $X$ such that the metric space $(X, d)$ is complete. Let $T : X \to X$ be a nondecreasing mapping. Suppose that the following assertions hold: (1) there exists $k \in [0, 1)$ such that $d(Tx, Ty) \leq kd(x, y)$ for all $x, y \in X$ with $x \ a \ y$; (2) there exists $x_0 \in X$ such that $x_0 \ a \ Tx_0$; (3) if $\{x_n\}$ is a nondecreasing sequence in $X$ such that $x_n \to x \in X$ as $n \to \infty$, then $x_n \ a \ x$ for all $n$. Then $T$ has a fixed point.
Sao et al. (2008), in this paper, authors have established common fixed point theorem using compatibility in fuzzy 2-metric space.

Bhaskar and Lakshmikantham (2006), in this paper, authors have introduced the concept of a coupled fixed point and the mixed monotone property.

Huang and Zhang (2007), in this paper, authors have described convergence in cone metric spaces, and introduced completeness. Then they proved some fixed point theorems of contractive mappings on cone metric spaces.

Nieto and Lopez (2007) in this paper, authors have extended the results obtained from Ran and Reurings (2004) for non-decreasing mappings and obtained a unique solution for a first order ordinary differential equation with periodic boundary conditions.

Ili and Rako (2008) and; Abbas and Jungck (2008), in these two papers, authors have addressed some common fixed point theorems were proved for maps on cone metric spaces. These papers introduce some basic topological concepts and definitions in cone metric spaces.

Mustafa et al. (2008), in this paper, authors have established fixed point theorems in $G$-metric spaces.

Dutta and Choudhury (2008), in this paper, authors have proposed that let $(X, d)$ be a complete metric space and $T : X \to X$ be a mapping satisfying $\psi(d(Tx,Ty)) \leq \psi(d(x, y)) - \phi(d(x, y))$, for all $x, y \in X$, where $\psi$ and $\phi$ are altering distance functions. Then $T$ has an unique fixed point.

Dorić (2009), in this paper, author has proposed that let $(X, d)$ be a complete metric space and $T : X \to X$ be a mapping satisfying $\psi(d(Tx,Ty)) \leq \psi(M(x, y)) - \phi(M(x, y))$, for all $x, y \in X$, where
M(x, y) = max \{ d(x, y), d(Tx, x), d(Ty, y), \frac{1}{2}[d(y, Tx) + d(x, Ty)] \}, where \( \psi \) is an altering distance function and \( \phi \) a lower semi-continuous function with \( \phi(t) = 0 \) if and only if \( t = 0 \). Then \( T \) has a unique fixed point.

**Harjani and Sadarangani (2010),** in this paper, authors have proposed that let \((X, \alpha)\) be a partially ordered set and suppose that there exists a metric \( d \) in \( X \) such that \((X, d)\) is a complete metric space. Let \( T: X \to X \) be a nondecreasing mapping such that \( \psi(d(Tx, Ty)) \leq \psi(d(x, y)) - \phi(d(x, y)) \), for all \( x, y \in X \) with \( x \alpha y \), where \( \psi \) and \( \phi \) are altering distance functions. Also suppose either (I) \( T \) is continuous or (II) If \( \{x_n\} \subset X \) is a nondecreasing sequence with \( x_n \to x \in X \), then \( x_n \alpha x \) for all \( n \).

If there exists \( x_0 \in X \) with \( x_0 \alpha Tx_0 \), then \( T \) has a fixed point.

**Choudhury and Maity (2011),** in this paper, authors have proposed that let \((X, \epsilon)\) be a partially ordered set such that \( X \) is a complete G-metric space, and \( F: X \times X \to X \) be a mapping having the mixed monotone property on \( X \). Suppose there exists \( k \in [0, 1) \) such that \( G(F(x, y), F(u, v), F(w, z)) \leq k/2 * (G(x, u, w) + G(y, v, z)). \) For all \( x, y, z, u, v, w \in X \) for which \( x \leq u \leq w \) and \( y \leq v \leq z \), where either \( u \neq w \) or \( v \neq z \). If there exists \( x_0, y_0 \in X \) such that \( x_0 \epsilon F(x_0, y_0) \) and \( y_0 \epsilon F(y_0, x_0) \) and either:

(a) \( F \) is continuous or

(b) \( X \) has the following property:

(i) if a non-decreasing sequence \( \{x_n\} \to x \), then \( x_n \epsilon x \) for all \( n \in N \),

(ii) if a non-increasing sequence \( \{y_n\} \to y \), then \( y_n \epsilon y \) for all \( n \in N \),

then \( F \) has a coupled fixed point.

**Jachymski (2011)** established a nice geometric lemma and proved that Theorem of Harjani and Sadarangani (2010) can be deduced from an earlier result of O’Regan and Petrusel (2008).

**Eslamian and Abkar (2012, in press),** in this paper, authors have proposed that let \((X, d)\) be a complete metric space and \( T: X \to X \) be a mapping satisfying \( \psi(d(Tx, Ty)) \leq \alpha(d(x, y)) - \beta(d(x, y)) \)
for all \( x, y \in X \), where \( \psi, \alpha, \beta : [0, \infty) \to [0, \infty) \) are such that \( \psi \) is an altering distance function, \( \alpha \) is continuous, \( \beta \) is lower semi-continuous, \( \alpha(0) = \beta(0) = 0 \) and \( \psi(t) - \alpha(t) + \beta(t) > 0 \) for all \( t > 0 \). Then \( T \) has a unique fixed point. Indeed, the contractive condition (1) can be written as: 
\[
\psi(d(Tx, Ty)) \leq \psi(d(x, y)) - \phi(d(x, y)),
\]
where \( \phi : [0, \infty) \to [0, \infty) \) is given by \( \phi(t) = \psi(t) - \alpha(t) + \beta(t) \), \( t \geq 0 \).