A Research Proposal

on

HIGH POWER LASER INTERACTION WITH PLASMAS AND SEMICONDUCTORS

Submitted to

LOVELY PROFESSIONAL UNIVERSITY

in partial fulfillment of the requirements for the award of degree of

DOCTOR OF PHILOSOPHY (Ph.D.) IN Physics

Submitted by: Vishal Thakur

Supervised by: Dr. Niti Kant

FACULTY OF TECHNOLOGY AND SCIENCES
LOVELY PROFESSIONAL UNIVERSITY
PUNJAB
INTRODUCTION

The recent advances in the high power laser technology have enabled experiments using laser pulses focused to extremely high intensity of the order of $10^{20}$ W/cm$^2$. These make possible the exploration of parameters in both atomic and plasma physics. The study of the phenomena related to self-focusing of intense laser light propagating in plasma has become a subject of considerable interest, since these phenomena play an important role in a large amount of high power laser applications, such as x-ray lasers, harmonic generation, laser driven plasma accelerators and fast igniter concept of inertial confinement fusion. For these applications performed laser channels are required to further guide the laser beam beyond the Rayleigh length, after which the beam expands infinitely in vacuum due to natural diffraction. Relativistic nonlinear optical effects are arisen that are most important phenomena to study such as harmonic generation, relativistic self-focusing, thermal self-focusing and ponderomotive self focusing.

The interaction of high intensity laser radiation with plasma is an extensively studied physical phenomenon. The study of self-focusing and various instabilities that arise due to propagation of intense laser pulses through plasma are of prime importance, since these phenomena significantly govern the experiments on advanced physical events such as inertial confinement fusion, x-ray lasers, optical harmonic generation and laser driven accelerators. When an intense laser beam propagates through partially ionized plasma it alters its refractive index, which now has a linear as well as intensity dependent nonlinear component. The free electrons of ionized plasma interact with the propagating laser radiation leading to ponderomotive and relativistic nonlinear effects, while the unionized atoms give rise to atomic nonlinearities.

When an intense laser radiation propagates through plasma the relativistic nonlinearities lead to self-focusing of the laser pulse when the laser power exceeds the critical power. Besides this, ponderomotive nonlinearity leading to plasma density perturbation is also expected to affect the focusing properties of the laser pulse. The intense laser beam sets the plasma electrons in relativistic quiver motion. Consequently, ponderomotive nonlinearity sets in, leading to electron density perturbation inside the plasma. This perturbation is caused due to $\mathbf{V}\times\mathbf{B}$ force that the radiation field exerts on the free plasma electrons. The effects of relativistic nonlinearity on the laser pulse as it
traverses through partially stripped plasma have been studied in great detail. However, the
effects of ponderomotive nonlinearity along with these effects have not been included.
Since both the nonlinearities after the propagation of a laser beam through plasma, so it is
important to study their combined effect.

The advent of high power lasers and consequent interest in laser induced fusion
generation renewed the vigorous interest in nonlinear laser plasma interactions, including
harmonic generation. Many phenomena, not considered important enough earlier because
of low available powers of electromagnetic beams, became very important in plasma.
With the advent of a very high-power source of electromagnetic radiation, the electron
velocity in plasma may become quite large (comparable to the light velocity in free
space). Hence, the effect of mass variation must be taken in to account. The relativistic
effect of an intense laser propagation in a plasma leads to self-focusing because the
dielectric constant of plasma in an increasing function of the intensity. The
ponderomotive force of the focused laser beam pushes the electrons out of the region of
high intensity, reduces the local electron density, and increase the plasma dielectric
function further, leading to even more self-focusing of the laser.

The interaction of lasers with semiconductors has been a fascinating field of
research for several decades. Semiconductors provide a compact and less expensive
medium to model nonlinear phenomena encountered in laser-produced plasmas. The
observation of self-focusing in semiconductors is of great relevance to the practical
applications and possibilities of optical limiting devices. In equilibrium, the temperature
of the free carriers is the same as that of the crystal so that the net energy exchange
between the carriers and the lattice of the crystal is zero. When an electric field is applied,
the free carriers gain energies, which causes the temperature to be higher than that of the
crystal in the steady state. For moderate values of the electric field, the increase in the
temperature of carriers is proportional to the square of the electric field. The change in
temperature of the carriers leads to corresponding change in the effective mass of carriers.
This effect is important for laser self-focusing in semiconductors. This high power laser
plasma interaction continues to be a front line area of research. The significant interest is
currently focused on laser interaction with atomic clusters. An intense short pulse laser
quickly converts them in to plasma balls which expand rapidly under hydrodynamic
expansion or coulomb explosion. As the electron density inside an expanding cluster
decreases and approaches thrice the critical density, the electron responses to the laser is
resonantly enhanced and one observes a host of exciting phenomena e.g. Strong absorption of laser energy, efficient generation of harmonics, self-focusing and energetic neutron production.

Most of the electromagnetic beams (e.g. Gaussian beams) have a non-uniform distribution of irradiance along the wave front, there was a need to take into account this non-uniformity in the theory of harmonic generation. It is well known that such beams exhibit the phenomenon of self-focusing/defocusing. Since for a given power of the beam, it is found that the average of the power of the electric vector in the wave-front is much higher for non-uniform irradiance distribution, than that for uniform irradiance distribution hence the magnitude of the generated harmonic is higher in the case of non-uniform irradiance. This provides a strong motivation for the study of the growth of the harmonics in a plasma taking self-focusing into account. The propagation of electromagnetic waves of non-uniform intensity distribution through plasmas is a problem of practical importance. At low power densities we have seen that diffraction causes divergence of the wave, however, at higher power densities this picture is changed drastically. In high intensity laser plasma interactions, a laser pulse beam can overcome natural diffracting defocusing and can remain focused via its own nonlinear interaction with the plasma.

The self-focusing effect can be produced due to the relativistic mass increase of electrons oscillating in a very high intensity laser field. The effective mass of plasma electrons decreases towards the axis of the laser pulse due to the ponderomotive force of the intense pulse, thus producing a maximum in the refractive index which gives rise to the formation of a self-focusing structure. This allows the propagation distance of the beam to be extended and enables the production of much higher peak intensities than could be achieved by focusing in vacuum. For a Gaussian intensity profile, self-focusing develops from this relativistic effect if the power of laser pulse exceeds a critical value; i.e. \( P_{cr} = 17 \left( \omega_0/\omega \right)^2 \text{GW} \). The self-focusing also occurs due to density perturbation caused by radiation or thermal pressure.
IMPORTANCE OF SELF FOCUSING:

The self-focusing of laser beam plays an important role in a large number of applications such as laser-driven fusion, laser-driven acceleration, x-ray lasers, optical harmonic generation, the production of quasi mono-energetic electron bunches, electron acceleration in wake-field, fast igniter concept of inertial confinement fusion etc. Self-focusing is very undesirable in laser fusion applications where it could prevent compression of fuel pellets. On the other hand the self-focusing of a laser beam into a filament would provide an effective method of achieving the high flux densities required to study laser plasma interactions such as generation of electron-positron pairs.

Self-focusing effect imposes a limit on the power that can be transmitted through an optical medium. It means self-focusing reduces the threshold for the occurrence of other nonlinear optical processes. Self-focusing often leads to damage in optical materials so it is a limiting factor in the design of high-power laser system but it can be harnessed for the design of optical power limiters and switches. The ponderomotive force associated with an intense laser beam expels electrons radially and can lead to cavitation in plasma. Relativistic effects as well as ponderomotive expulsion of electrons modify the refractive index. For laser power exceeding the critical power, the analysis of relativistic self-focusing indicates that a significant contraction of the spot size and a corresponding increase in intensity is possible.

In self-focusing of electromagnetic beams in collisional plasmas with nonlinear absorption, the nonlinearity in absorption tends to cancel the effect of divergence on account of diffraction. The beam-width and attenuation depends on distance of propagation. The fact that the relativistic mechanism is the only mechanism of self-focusing, which can manifest itself for intense pico-second laser pulse, makes the study very important. It could produce ultra-high laser irradiance exceeding $10^{19}$-$10^{20}$ W/cm$^2$ over distance much greater than the Rayleigh length determined by natural diffraction. In terms of self-focusing of a laser pulse in plasma with periodic density ripple, the laser induces modifications of the plasma refractive index via relativistic and ponderomotive nonlinearities. In the region of high plasma density, the self-focusing effect suppresses the diffraction, divergence, and the laser converges. As the beam enters in to the low density region, the diffraction tends to diverge it, offsetting the convergence due to curvature it has acquired. For a given set of plasma parameters, there is a critical power of the laser
above which it propagates in a periodically focused manner. Below this power laser undergoes overall divergence. At substantially higher powers, the laser beam continues to converge until the saturation effect of nonlinearity suppresses the self-focusing and diffraction predominates. The effect of density ripple is to cause overall increase in the self-focusing length. The minimum spot size decreases with the wave number of the ripple. In case of ponderomotive self-focusing of a short laser pulse in an under-dense plasma density ramp, the pulse may acquire a minimum spot size due to the ponderomotive self-focusing. The self-focused laser pulse diffracts and focuses periodically because of the mismatch between the channel size and spot size. For a given laser spot size, the oscillation amplitude becomes larger for a higher plasma density due to the enhanced relativistic effect.

The relativistic effect of high power laser beam propagation in plasma produces self-focusing by relativistic mass change. If in plasma, the frequency exceeds the natural frequency of the electron oscillations, then the electrons will be forced out of the beam field. The focused laser beam exerts a radial ponderomotive force on electrons and expels them outward of the beam, producing a lower density in the centre regions which results in focusing the radiation. Remarkable self-focusing effects have been observed recently with femtosecond laser beams propagation in the atmosphere: light filaments in the air with repeated filamentation over distances greater than 10 km have been observed. A broad (ultraviolet to mid-infrared ) spectrum of radiation is generated from such air filaments, permitting spectroscopy and localized remote sensing of chemical species and aerosols in the lower atmosphere. The entire absorption spectrum can be determined by a single pulse from a portable femtosecond laser. Another exciting possibility of the use of these filaments containing plasma filaments is to guide lightning away from sensitive sites.

**IMPORTANCE OF PLASMA DENSITY RAMP:**

The plasma density ramp plays an important role during laser-plasma interaction. It is known that a high-power laser beam propagating through under dense plasma can acquire a minimum spot size due to relativistic self-focusing. After the focusing, the nonlinear refraction starts weakening and the spot size of the laser increases, showing periodic self-focusing/defocusing behaviour with the distance of propagation. To reduce the defocusing effect, a localized upward plasma density ramp is introduced, so that the
laser beam obtains a minimum spot size and maintains it with only a mild ripple. The density ramp could be important for the self-focusing of a high power laser by choosing the laser and plasma parameters. With the increase in plasma density, the self-focusing effect becomes stronger. It is because as the laser propagates through the density ramp region, it sees a slowly narrowing channel. In such an environment, the oscillation amplitude of the spot size shrinks, while its frequency increases. Also as the equilibrium electron density is an increasing function of the distance of propagation of the laser, the plasma dielectric constant decreases rapidly as the beam penetrates deeper and deeper into the plasma. Consequently, the self-focusing effect is enhanced and the laser is more focused. However, the minimum plasma density is chosen in the assumption of under dense plasma. The length of plasma density ramp is considered to avoid the maximum defocusing of the laser and better focusing is observed by increasing the length of the density ramp. But the plasma density should not be much larger; otherwise, the laser can be reflected because of the over dense plasma effect. So, plasma density ramp plays an important role to make the self-focusing stronger. This kind of plasma density ramp may be observed in a gas jet plasma experiment.

In the propagation of a high power Gaussian laser beam through plasma with a density ramp where a magnetic field is present, the combined role of the plasma density ramp and the transverse magnetic field is found to enhance the laser beam focusing in plasma. The spot size of the laser beam shrinks as the beam penetrates into the plasma due to the role of the plasma density ramp. The magnetic field acts as a catalyst for self-focusing of a laser beam during propagation in plasma density ramp. By using this scheme, the laser not only becomes focused but it can also propagate over a long distance without divergence, which is a basic requirement for many laser-driven applications.

**REVIEW OF LITERATURE:**

The process of self-focusing of laser beams in plasma is of great importance for various laser-plasma interactions and laser fusion schemes etc. So laser beams have always been an interesting area of research for many years.

The first discussion of self-focusing of laser beams in plasma was published by *Askaryan (1962)*. He considered the energy momentum flux density of the laser beam at self-focusing, where the whole plasma has been expelled and where the pressure is balanced by the plasma pressure profile acting against the centre of the laser beam.
Askaryan was able to compare the necessary optical intensities for compensating the gas dynamic pressure.

_Sodha et al. (1971)_ investigated theoretically the propagation and focusing of an electromagnetic wave in inhomogeneous dielectrics. They concluded that the focusing length is enhanced in a medium where the dielectric constant is a decreasing function of axial distance of propagation and vice-versa.

_Sodha et al. (1974)_ investigated the self-focusing of a cylindrically symmetric Gaussian electromagnetic pulse in collision-less and coalitional plasmas by considering the ponderomotive force and the non-uniform heating (and the consequent redistribution of electrons) as the sources of non-linearity. They considered that the duration of pulse is larger than the characteristic time of non-linearity. They found that the beam is focused in a moving filament. Because of relaxation effects the peak of the pulse is shifted to higher values in case of coalitional plasmas and the pulse is severely distorted because of self-focusing; the shift of peak in the case of collision-less plasmas is not significant.

_Liu and Tripathi (2000)_ investigated the laser frequency up-shift, self-defocusing and ring formation in tunnel ionizing gases and plasmas. In their work the combined effects of tunnel ionization of gases on laser frequency up-shift, self-defocusing and ring formation are considered self-consistently. A high-intensity short pulse laser causes rapid tunnel ionization of a gas and the increasing plasma density leads to a decreasing refractive index, modulating the phase of the laser as it propagates and causing frequency up-shift and super-continuum generation. For laser intensity profile peaking on axis, the tunnel ionization produces a minimum of refractive index on axis, thus defocusing the laser. The defocusing reduces the ionization rate and frequency up-shift. It was found by them that as the laser propagates over a Rayleigh length its trailing portion develops a ring shape distribution due to stronger defocusing of rays on axis than the off-axis rays.

_Akozbek et al (2002)_ have got third harmonic generation and self channeling in air using high power femtosecond laser and showed both theoretically and experimentally, that during laser filamentation in air, an intense ultra short third harmonic is generated forming two colored filament. They experimentally observed pump pulses and third harmonic generation with Kerr effects and formulate self consistent and complete set of non-linear Schrödinger equations for a pair of coupled beams-
fundamental and its third harmonic. Yang et al (2003) observed strong third harmonic emission with a conversion efficiency higher than 0.1% from plasma channel formed by self-guided femtosecond laser pulses propagating in air. They found an optimized condition under which third harmonic conversion efficiency is maximized. Their experimental results shows that radiation of emission in ultraviolet wavelength range makes a major attribution to third harmonic emission, whereas the effects of self-phase modulation are not important when intense laser pulse interacts with gaseous media. The model has exact solution in which third harmonic beam takes a Gaussian profile. For a phase mismatch, that is zero or negative.

Faure et al. (2002) discussed the effects of pulse duration on self-focusing of ultra-short lasers in underdense plasma. It was shown experimentally by them that the critical power for relativistic self-focusing $P_c$ is not the only relevant parameter, in particular when the laser pulse duration is comparable to plasma particle motion times: $\omega_p^{-1}$ for electrons and $\Omega_{pi}^{-1}$ for ions. Using time resolved shadowgraphs, it was demonstrated by them: (1) a pulse does not relativistically self-focuses if its duration is too short compared to $\omega_p^{-1}$, even in the case where the power is greater than $P_c$. This is due to defocusing by the longitudinal wake which is generated by the laser pulse itself. (2) For pulses longer than $\Omega_{pi}^{-1}$, self-focusing can occur even for powers lower than $P_c$. This is due to the radial expansion of ions, creating a channel whose effect combines with relativistic focusing and helps the pulse to self-focus.

Esarey et al. (1993) have observed the second harmonic in the presence of density gradients. This is due to the laser-induced quiver motion of the electrons across a density gradient at the laser frequency, Pallavi et al (2007). In second harmonic process, (two or more) photons of energy $h\omega_1$, and momentum $hk_1$, each combine to generate a photon of second harmonic radiation of energy $h\omega_2$ and momentum $hk_2$, where $\omega_1$, $k_1$ and $\omega_2$, $k_2$ are the frequencies and wave numbers of fundamental and second harmonic respectively, which satisfy the linear dispersion relation for electromagnetic waves. Energy and momentum conservation in a second harmonic process demands $\omega_2 = 2\omega_1, k_2 = 2k_1$. As plasmas are dispersive media, so these conditions are quite restrictive, hence the above conditions are not satisfied. Langmuir wave couples with the laser to produce efficient second harmonic generation, Matsumoto and Tanaka (1995). Also process could be
important only when additional momentum \( \hbar k_2 - 2\hbar k_1 \) is available per photon in the system.

**Nitikant and Sharma (2004)** have studied the resonant second harmonic generation of a short pulse laser in plasma channel and analyzed the generation of second harmonic in plasma in the presence of a magnetic wiggler. They have seen the effect of self-focusing on resonant second harmonic generation under wiggler magnetic field. Wiggler magnetic field provides the additional angular momentum to the second harmonic photon to make the process resonant. It also plays an important role in the enhancement of the intensity of second harmonic wave. Ferrante and Zarcone (2004) have obtained harmonic generation in the skin layer of hot dense plasma and found the explicit dependencies of the third harmonic generation efficiency on the plasma and pump field parameters.

A theory is developed for ultra-relativistic laser-plasmas. It is used to compare and optimize possible regimes of three-dimensional wake field acceleration. The optimal scalings for laser wake field electron acceleration are obtained analytically, Pukhov and Gordienko (2006). High-order harmonic generation due to the interaction of a short ultra relativistic laser pulse with overdense plasma is studied analytically and numerically. On the basis of ultra relativistic similarity theory we show that the high-order harmonic spectrum is universal, i.e., it does not depend on the interaction details. It has been shown that managing time-dependent polarization of the relativistically intense laser pulse incident on a plasma surface allows us to gate a single attosecond x-ray burst even when a multicycle driver is used. The single x-ray burst is emitted when the tangential component of the vector potential at the plasma surface vanishes, Baeva et al (2006).

Nonlinear focusing of an intense laser beam propagating in a plasma channel is studied by means of the variational method including the effects of relativistic self-focusing (RSF), ponderomotive self-channeling (PSC), and higher-order relativistic nonlinearity (HRN). The effect of HRN defocuses the beam and has the same order of magnitude in the spot size evolution equation as that of PSC. The beam focusing singularity for laser power exceeding the critical power for RSF is prevented in the presence of HRN. And expression for the matched power is calculated, Liu et al (2006). Surface acceleration of fast electrons in intense laser-plasma interactions has been reviewed. When preformed plasma is presented in front of a solid target with higher laser
intensity, the emission direction of fast electrons is changed to the target surface direction from the laser and specular directions. This feature could be caused by the formation of a strong static magnetic field along the target surface which traps and holds fast electrons on the surface. It has been found that the increase in the laser intensity due to relativistic self-focusing in plasma plays an important role for the formation. The strength of the magnetic field is calculated from the bent angle of the electrons, resulting in tens of percent of laser magnetic field, which agrees well with a two-dimensional particle-in-cell calculation. The strong surface currents explain the high conversion efficiency on the cone-guided fast ignitor experiments, Habara (2006).

The effects of relativistic nonlinearities on stimulated Raman scattering (SRS) of mildly relativistic laser radiation propagating through pre-ionized underdense plasma is studied. The nonlinear mode coupling equations and linearized growth rates of Raman instabilities in presence of combined effects of relativistic and ponderomotive nonlinearities are derived. The temporal evolution of SRS process is then studied and analyzed using numerical techniques in presence of the two nonlinearities. It is observed that relativistic effects now taken into account significantly affect the growth rates and the nonlinear evolution of SRS process as compared to the case where they are neglected. The study shows that inclusion of relativistic nonlinearities along with the ponderomotive nonlinearities destabilizes SRS process, Jha et al (2006).

The ability to select and stabilize a single filament during propagation of an ultra-short high intensity laser pulse in air makes it possible to examine the longitudinal structure of the plasma channel left in its wake. The detailed measurements of plasma density variations along laser propagation have been presented by Eisenmann et al (2007). The coherent optical transition radiation emitted by an electron beam from laser-plasma interaction has been measured. The measurement of the spectrum of the radiation reveals fine structure of the electron beam in the range 400-1000nm. These structures are reproduced using an electron distribution from a 3D particle-in-cell simulation and are attributed to microbunching of the electron bunch due to its interaction with the laser field, Glinec et al (2007).

The propagation of femtosecond, multiterawatt, relativistic laser pulses in transparent plasma is studied by Liu et al (2007). The focusing properties of the beam are shown to be governed by the laser power as well as the laser intensity. An increase in the laser intensity, the emission direction of fast electrons is changed to the target surface direction from the laser and specular directions. This feature could be caused by the formation of a strong static magnetic field along the target surface which traps and holds fast electrons on the surface. It has been found that the increase in the laser intensity due to relativistic self-focusing in plasma plays an important role for the formation. The strength of the magnetic field is calculated from the bent angle of the electrons, resulting in tens of percent of laser magnetic field, which agrees well with a two-dimensional particle-in-cell calculation. The strong surface currents explain the high conversion efficiency on the cone-guided fast ignitor experiments, Habara (2006).
intensity leads to an enhancement of ponderomotive self-channeling but a stronger weakening of relativistic self-focusing. The oscillations of the beam spot size along the propagation distance come from the variability of the focusing force in terms of the laser intensity; and the dependence on the laser intensity is negligible in the weakly relativistic limit.

An experimental observation suggesting plasma channel formation by focusing a relativistic laser pulse into long-scale-length preformed plasma has been reported. The channel direction coincides with the laser axis. Laser light transmittance measurement indicates laser channeling into the high-density plasma with relativistic self-focusing. A three-dimensional particle-in-cell simulation reproduces the plasma channel and reveals that the collimated hot-electron beam is generated along the laser axis in the laser channeling. These findings hold the promising possibility of fast heating dense fuel plasma with a relativistic laser pulse, Lei et al (2007).

Dahiya et al (2007) have calculated the efficiency of phase-matched second and third harmonic generation in plasmas with density ripple. Nitikant and Sharma (2004) have seen the effect of pulse slippage on a resonant second harmonic generation of a short pulse laser in plasma by application of magnetic wiggler. The group velocity of second harmonic wave is greater than that of fundamental wave, and hence generated pulse slips out of the domain of the fundamental laser pulse and its amplitude saturates with time. The generation of plasma wave and third harmonic generation in hot collision less plasma by a Gaussian ultra intense laser beam, when relativistic and ponderomotive nonlinearities are operative has been investigated. The dynamical equation for the pump laser beam when these two nonlinearities are operative has been derived. The solution of pump laser beam has been obtained within the paraxial ray approximation.

The effects of the interaction of an intense femtosecond laser pulse with large atomic clusters are considered. The pulse intensity is of the order of $10^{18} \text{Wcm}^{-2}$. New effects appear when the magnetic component of the Lorentz force is taken into account, Rastunkov (2007). Numerical computations have been made for linear inhomogeneity and saturating nonlinearity, characteristic of dielectrics and collisional plasmas. The maximum and minimum of the beam width keep decreasing with increase in distance of propagation (or absorption), till the beam becomes very weak and diverges steeply; penetration in an overdense medium also decreases with increasing absorption.
The theoretical and numerical analysis show that high power microwave can generate harmonic in the plasma filled waveguide. The most promising approach to increase the beam current density is introducing background plasma. By this means, the beam space charge can be fully or partially neutralized, the beam current may increase greatly and axial guide magnetic fields could be lowered, the size, weigh and cost of the devices could be reduced, Fu and Yan (2007). In high power plasma filled microwave devices, the power of microwave may be very strong, sometimes over 1 GW. In this case, the electric field of electromagnetic wave is very strong, many nonlinear phenomena may be induced, such as self-focusing and self-guiding, ponderomotive effects, wake-field, solution, harmonic generation. Because of the complexity of phenomena, the physical process in the devices is far from fully understood yet.

It has been shown that both the maximum energy gain and the accelerated beam quality can be efficiently controlled by the plasma density profile. Choosing a proper *between the bunch of relativistic particles and the plasma wave over extended distances, Pukhov and Kostyukov (2008).

Faisal et al. (2008) investigated self-focusing of electromagnetic pulsed beams in collisional plasma by using paraxial approximation. They developed the energy balance equation for electrons and equations which express the equality of pressure gradient of electrons and ions to the force due to space charge field. They also solved equation for beam width parameter * for initial time profile at z=0 of the pulse to obtain * as a function of normalized distance and time profile. It is seen by them that in the initial period the beam suffers steady divergence since the nonlinearity does not build up to sufficient extent. Later, the behaviour changes to oscillatory divergence, then oscillatory convergence, and again oscillatory divergence and finally steady divergence. This is explained by the fact that focusing is dependent on the rate of change of nonlinearity with the irradiance, rather than on the magnitude of the nonlinearity. They used both Gaussian as well as since time profile of the pulse for investigation.
OBJECTIVES


ABSTRACT

In the presence of a magnetic wiggler of suitable period, a Gaussian laser beam resonantly generates a second harmonic in plasma. The phase matching conditions for the process are satisfied for a specific value of the wiggler period. The relativistic self-focusing of the fundamental pulse enhances the intensity of the second-harmonic pulse. The harmonic undergoes periodic focusing in the plasma channel created by the fundamental wave. The normalized second-harmonic amplitude varies periodically with distance with successive maxima acquiring higher values. In this problem we will be discussing the effect of relativistic self-focusing on second harmonic generation in plasma under plasma density ramp. We will solve the differential equations for the beam width parameters and the equation for second harmonic amplitude under paraxial approximation in Mathematica software. We will discuss the results for different values of plasma density. We will also see how the relativistic self-focusing is affected by taking into account the plasma density ramp.

THEORETICAL CONSIDERATIONS

Let us consider the dielectric constant of the form,

\[ \varepsilon = \varepsilon_0 + \Phi(EE') \]  

(1)

With \( \varepsilon_0 = 1 - \omega_b^2 / \omega^2 \), \( \omega_b^2 = 4\pi n(\xi)e^2 / m \) and \( m = m_0 / \sqrt{1 - v^2 / c^2} \) or \( m = m_0 \gamma \)

Where \( \gamma = 1 / \sqrt{1 - v^2 / c^2} \) and \( n(\xi) = n_0 \tan(\xi / d) \)

Therefore \( \varepsilon_0 = 1 - (\omega_{\rho0}^2 / \gamma^2)\tan(\xi / d) \), where \( \omega_{\rho0}^2 = 4\pi n_0 e^2 / m_0 \)

And

\[ \omega_{\rho}^2 = (4\pi n_0 e^2 / m_0 \gamma)\tan(\xi / d) = (\omega_{\rho0}^2 / \gamma)\tan(\xi / d) \]  

(2)
Now, in case of collision-less plasma, the nonlinearity in the dielectric constant is mainly due to ponderomotive force and the nonlinear part of dielectric constant is given by

$$\Phi(EE^*) = \frac{\omega_{\text{p}}^2}{\omega^2} \tan \left( \frac{\xi}{d} \right) \left[ 1 - \exp \left( -\frac{3m_0\gamma}{4M} \alpha EE^* \right) \right]$$

(3)

**Relativistic Self-focusing:**

We have eqn.

$$\frac{\partial^2 \overline{E}}{\partial x^2} + \frac{\partial^2 \overline{E}}{\partial y^2} + \frac{\partial^2 \overline{E}}{\partial z^2} + \left( \frac{\omega^2}{c^2} \right) c \overline{E} = 0$$

(4)

This equation is solved by employing Wentzel-Kramers-Brillouin (WKB) approximation.

Also, we know that $k = (\omega / c) \varepsilon_0^{1/2}$, $k = (\sqrt{\omega^2 - (\omega_{\text{p}}^2 / \gamma) \tan(z/dR_\gamma)}) / c$

Therefore from eqn. (3.9) we get

$$\overline{E} = A(x, y, z) \exp \left[ i \left( \omega t - \frac{z}{c} \sqrt{\omega^2 - \frac{\omega_{\text{p}}^2 \tan(z/dR_\gamma)}{\gamma}} \right) \right]$$

(5)

Where $\xi = z / R_\gamma$; $R_\gamma = k\xi^2$ is the diffraction length and $A(x, y, z)$ is the complex amplitude of electric field.

To solve equation (4) we express $A(x, y, z)$ as

$$A(x, y, z) = A_{\text{vp}}(x, y, z) \exp \left[ -iks(x, y, z) \right]$$

Or

$$A(x, y, z) = A_{\text{vp}}(x, y, z) \exp \left[ -i \frac{S \sqrt{\omega^2 - \frac{\omega_{\text{p}}^2 \tan(z/dR_\gamma)}{\gamma}}}{c} \right]$$

(6)

Where ‘$A_{\text{vp}}$’ and ‘$S$’ are real functions of $x$, $y$ and $z$. After solving the Eq. (4) and then equate the real parts and imaginary parts, we get
Real part

\[
\frac{1}{\omega^2 A_{\lambda p}} \left[ \frac{\partial^2 A_{\lambda p}}{\partial x^2} + \frac{\partial^2 A_{\lambda p}}{\partial y^2} \right] + \Phi A_{\lambda p}^2 = \frac{1}{\omega^2 c^2} \left( \omega^2 - \omega_{p_0}^2 \frac{\tan (z / dR_d)}{\gamma} \right) \left[ \frac{\partial S}{\partial x} + \left( \frac{\partial S}{\partial y} \right)^2 \right]
\]

\[
+ \frac{1}{\omega^2 c^2} \left( \frac{\partial S}{\partial z} \right) \left( \omega^2 - \omega_{p_0}^2 \frac{\tan (z / dR_d)}{\gamma} \right) - \frac{\omega_{p_0}^2 \sec^2 (z / dR_d)}{2\gamma dR_d} + \frac{\omega_{p_0}^4}{\omega^2 c^2} \left( \frac{\sec^2 (z / dR_d)}{2\gamma} \right)
\]

\[
- \frac{\omega_{p_0}^2 \sec^2 (z / dR_d)}{2\omega^2 c^2 \gamma dR_d} \left[ \frac{1}{\gamma dR_d} \left( \omega^2 - \omega_{p_0}^2 \frac{\tan (z / dR_d)}{\gamma} \right) \right]
\]  

(7)

Imaginary part

\[
\left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right) A_{\lambda p}^2 + \left( \frac{\partial A_{\lambda p}}{\partial x} \right) \left( \frac{\partial S}{\partial x} \right) + \left( \frac{\partial A_{\lambda p}}{\partial y} \right) \left( \frac{\partial S}{\partial y} \right) + \left[ \frac{1}{2} - \frac{\omega_{p_0}^2 \sec^2 (z / dR_d)}{4\gamma dR_d} \left( \omega^2 - \omega_{p_0}^2 \frac{\tan (z / dR_d)}{\gamma} \right) \right]
\]

\[
- \frac{A_{\lambda p} \omega_{p_0} \sec^2 (z / dR_d)}{\gamma dR_d} \left[ 1 + \frac{z}{dR_d} \frac{\tan (z / dR_d)}{\gamma} \right] = 0
\]

(8)

The solutions of equations (7) and (8) can be written as:

\[
S = \frac{x^2}{2} \beta_1 (z) + \frac{y^2}{2} \beta_2 (z) + \Phi (z)
\]

(9)

\[
A_{\lambda p}^2 = \frac{E_0^2}{f_1 (z) f_2 (z)} H_0 \left( \frac{\sqrt{2} x}{r_0 f_1 (z)} \right) H_p \left( \frac{\sqrt{2} y}{r_0 f_2 (z)} \right) \exp \left[ - \left( \frac{x^2}{r_0 f_1^2 (z)} + \frac{y^2}{r_0 f_2^2 (z)} \right) \right]
\]

(10)
Now using eqn. (7), (9) & (10) and equating the coefficients of $x^2$ & $y^2$ we get,

$$\frac{2}{f_1^2} \left( 1 - \frac{\omega_{p0}^2}{\gamma \omega^2} \tan \left( \frac{\xi}{d} \right) \right) - \frac{3m_e\alpha E_0^2}{2M} \left( \frac{r_{p0} \omega}{c} \right)^2 \left( 1 - \frac{\omega_{p0}^2}{\gamma \omega^2} \tan \left( \frac{\xi}{d} \right) \right) \left( \frac{\omega_{p0}^2}{\omega^2} \right) \tan \left( \frac{\xi}{d} \right) \exp \left[ - \frac{3m_e\alpha \gamma E_0^2}{4M f_1 f_2} \right] = 0$$

$$\left[ 1 - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\tan \left( \frac{\xi}{d} \right)}{\gamma} - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\xi \sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d} \right] \frac{\partial^2 f_i}{\partial \xi^2}$$

$$+ \left[ 1 - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\tan \left( \frac{\xi}{d} \right)}{\gamma} + \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\xi \sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d} \right] \frac{1}{f_i} \left( \frac{\partial f_i}{\partial \xi} \right)^2$$

$$\left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d}$$

$$\left[ 1 - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\tan \left( \frac{\xi}{d} \right)}{\gamma} - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\xi \sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d} \right] \frac{\partial^2 f_i}{\partial \xi^2}$$

$$+ \left[ 1 - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\tan \left( \frac{\xi}{d} \right)}{\gamma} + \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\xi \sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d} \right] \frac{1}{f_i} \left( \frac{\partial f_i}{\partial \xi} \right)^2$$

$$\left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d}$$

(11)

And

$$\frac{2}{f_2^2} \left( 1 - \frac{\omega_{p0}^2}{\gamma \omega^2} \tan \left( \frac{\xi}{d} \right) \right) - \frac{3m_e\alpha E_0^2}{2M} \left( \frac{r_{p0} \omega}{c} \right)^2 \left( 1 - \frac{\omega_{p0}^2}{\gamma \omega^2} \tan \left( \frac{\xi}{d} \right) \right) \left( \frac{\omega_{p0}^2}{\omega^2} \right) \tan \left( \frac{\xi}{d} \right) \exp \left[ - \frac{3m_e\alpha \gamma E_0^2}{4M f_1 f_2} \right] = 0$$

$$\left[ 1 - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\tan \left( \frac{\xi}{d} \right)}{\gamma} - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\xi \sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d} \right] \frac{\partial^2 f_2}{\partial \xi^2}$$

$$+ \left[ 1 - \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\tan \left( \frac{\xi}{d} \right)}{\gamma} + \left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\xi \sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d} \right] \frac{1}{f_2} \left( \frac{\partial f_2}{\partial \xi} \right)^2$$

$$\left( \frac{\omega_{p0}^2}{\omega^2} \right) \frac{\sec^2 \left( \frac{\xi}{d} \right)}{2\gamma d}$$

(12)

Equations (11) and (12) are the expressions for beam width parameters $f_1$ and $f_2$ respectively.
Relativistic self-focusing of the main pulse and nonlinear second-harmonic current density:

Consider the propagation of intense Gaussian laser short pulse in a preformed plasma channel in the presence of a wiggler magnetic field $\vec{B}_w$:

$$\vec{E}_i = \hat{x}A_i(z,t) \exp[-i(\omega_i t - k_i z)],$$

$$A_i^2 = \frac{A_{i0}^2}{f_i^2(z)} \exp\left[-\frac{r^2}{f_i^2} \right],$$

$$\vec{B}_i = \frac{c\vec{k}_i \times \vec{E}_i}{\omega_i},$$

$$\vec{B}_w = \hat{y}B_0 \exp(ik_0 z),$$

Following Nitikant et al, the nonlinear current density at the second harmonic is

$$J_{2s}^NL = \frac{n_0 e^4 B_w E_i^2}{4ic\omega_i^2 m^\nu(\omega_i + iv)} \left(\frac{3k_i}{4\omega_i} + \frac{k_i + k_0}{\omega_i + iv}\right) \hat{z}.$$  \hspace{1cm} (14)

Now, the wave equation for the second-harmonic field $\vec{E}_2$ can be deduced from Maxwell’s equations,

$$\nabla^2 \vec{E}_2 = \frac{4\pi}{c^2} \frac{\partial \vec{J}_2}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \vec{E}_2}{\partial t^2}.$$ \hspace{1cm} (15)

In plasma the equation (4) reduces to,

$$\nabla^2 \vec{E} - \frac{\varepsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{\omega_p^2}{c^2} \vec{E}.$$ \hspace{1cm} (16)

The nonlinear plasma permittivity in the presence of the laser, for relativistic mass nonlinearity, may be written as

$$\varepsilon = \varepsilon_0 + \phi(E_i E_i^*)$$ \hspace{1cm} (17)
where \( \varepsilon_0 = 1 - \omega_p^2 / \omega_1^2 \) and \( \phi \) may be written as

\[
\phi = \frac{\omega_p^2}{\omega_1^2} \left[ 1 - \frac{1}{\left( 1 + \frac{1}{2} a^2 \right)^{1/2}} \right].
\]

Therefore eqn. (16) becomes,

\[
\left( \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \tilde{E} + \frac{1}{c^2} \left[ \varepsilon_0 + \phi(EE^*) \right] \mu^2 \tilde{E} = \frac{1}{c^2} \left( \frac{4\pi m_0 e^2}{m} \right) \tilde{E}
\]

Now, \( A = A_0 e^{ikS} \)

Where, \( A_0 \) and \( S \) are real functions of \( r \) and \( z \).

Now after solving eqn.(19) and equating real and imaginary parts we get,

Real Part:

\[
-2A_0 k^2 \frac{\partial S}{\partial z} - A_0 k^2 \left( \frac{\partial S}{\partial r} \right)^2 + \frac{\partial^2 A_0}{\partial r^2} + \frac{1}{c^2} \frac{\partial A_0}{r \partial r} + \frac{\omega^2}{c^2} \phi(EE^*) A_0 = \frac{4\pi m_0 e^2}{mc^2} A_0
\]

Imaginary Part:

\[
\frac{\partial A_0}{\partial z} + \frac{A_0}{2} \frac{\partial^2 S}{\partial r^2} + \frac{A_0}{r} \frac{\partial S}{\partial r} + \frac{A_0}{2r} \frac{\partial S}{\partial r} = 0
\]

Therefore, the equation governing the beam width parameter \( f_1 \) is given by

\[
\frac{d^2 f_1}{d z^2} = \frac{1}{f_1^3} - \frac{A_{10}^2 R_d^2 \varepsilon_2 \varepsilon_8}{2 \varepsilon_0 f_0^2 f_1^3} \exp \left\{ - \frac{\varepsilon_1 A_{10}^2}{2 \varepsilon_0 f_1^2} \right\},
\]

where \( R_d = k_i r_0^2 \) is the diffraction length. From equation (4) we may write

\[
\nabla^2 \tilde{E}_2 = \left[ \frac{4\omega_1^2 - 5\omega_p^2}{c^2} \right] \tilde{E}_2 = -\frac{8\pi i \omega_p}{C^2} \tilde{J}_2^{NL}.
\]
The complementary solution of this equation is

\[ E_{21} = A_2 \exp\{-ik_2S_2\} \exp[-i(2\omega_1 t - k_2 z)], \tag{24} \]

Where \( S_2 \) is a function of \( r \) and \( z \). Using equations (23) and (24), and equating real and imaginary parts,

\[
2 \frac{\partial^2 S_2}{\partial z^2} - \left( \frac{\partial S_2}{\partial r} \right)^2 + \frac{1}{k_2^2 A_2} \left( \frac{\partial^2 A_2}{\partial r^2} + \frac{1}{r} \frac{\partial A_2}{\partial r} \right) - 1 + \frac{4\omega_1^2 - 5\omega_2^2}{k_2^2 c^2} + \frac{4\omega_2^2}{\varepsilon_0 k_2^2 c^2} \phi(E_i E_i^*) = 0
\]

\[
- \frac{\partial A_2}{\partial z} \frac{\partial S_2}{\partial z} + \frac{\partial A_2}{\partial r} \frac{\partial S_2}{\partial r} - \frac{\partial A_2}{\partial r} \frac{\partial S_2}{\partial r} - A_2 \left( \frac{\partial^2 S_2}{\partial r^2} + \frac{1}{r} \frac{\partial S_2}{\partial r} \right) = 0
\]

In the paraxial ray approximation, \( r^2 << r_0^2 f_1^2 \), we expand \( S_2 \) as \( S_2 = \phi(z) + \beta_2^{-1} r^2 / 2 \), where \( \beta_2^{-1} \) represents the radius of curvatures of the wave front of the second-harmonic wave. For an initially Gaussian beam, we may write

\[ A_2 = \frac{A_{10}^2}{f_2^2(z)} \exp \left[ -\frac{2r^2}{r_0^2 f_2^2} \right]. \]

Substituting the values of \( S_2 \) and \( A_2 \) in equations (25) and (26) and equating the coefficients of \( r^2 \) on both sides, we obtain

\[ \beta_2 = \frac{1}{f_2} \frac{df_2}{dz}, \tag{27} \]

\[
\frac{d^2 f_2}{dz^2} = \frac{4}{k_2^2 r_0^4 f_2^3} - \frac{2\omega_1^2 A_{10}^2 f_2}{\varepsilon_0^2 c^2 k_2^2 r_0^2 f_1^4} \phi\left( \frac{A_{10}^2}{2f_1^2} \right),
\]

where \( \phi\left( \frac{A_{10}^2}{2f_1^2} \right) = \left( \varepsilon_s / \varepsilon_c \right) \exp\left( -\varepsilon_2 A_{10}^2 / 2 \varepsilon_0 f_1^2 \right). \)

Now let us suppose the solution (Particular Integral) of equation (23) is,

\[ E_{22} = A_2 \exp \{-i(2\omega_1 t - (2k_1 + k_0)z)\}, \tag{29} \]
where $A_2' = A_{20}'(z)\psi_2, \psi_2 = \exp[-r^2/r_0^2 f_1^2] \exp[-2ik_1 S_1]$. 

Using above equation’s, we obtain

$$4ik_1 \psi_2 \frac{\partial A_{20}}{\partial z} + \left[ \frac{4\omega_1^2 - 5\omega_p^2}{c^2} + \frac{4\omega_1^2}{c^2} \phi(E^*_1 E^*_1) - (2k_1 + k_0)^2 \right] \times A_{20} \psi_2 + A_{20} \frac{\partial^2 \psi_2}{\partial r^2} + A_{20} \frac{1}{r} \frac{\partial \psi_2}{\partial r}$$

$$= -\frac{\omega_p^2 e^2 B_w A_{10}^2}{2c^3 \omega_1 m^2 (\omega_i + i\nu) f_1^2} \left[ \frac{3k_1}{4\omega_1} + k_1 + k_0 \right] \exp \left[ \frac{-r^2}{r_0^2 f_1^2} \right]$$

(30)

Multiplying the above equation by $\psi_2^* r dr$ and integrating with respect to r, we get

$$\frac{\partial A_{20}^*}{\partial \xi} + \left[ \frac{4\omega_1^2 - 5\omega_p^2}{c^2} \right] R_d + \frac{4\omega_1^2 R_d}{c^2} \left[ \frac{\omega_p^2}{\omega_1} \left[ 1 - \frac{1}{\left( 1 + \frac{1}{2} a^2 \right)^{1/2}} \right] \right] - (2k_1 + k_0)^2 R_d$$

$$\left[ \frac{1}{4ik_1} - \frac{1}{2if_1^2} \right] A_{20}^*$$

(31)


**ABSTRACT**

A Gaussian laser-beam resonantly generates a third-harmonic wave in plasma in the presence of a wiggler magnetic-field of suitable period. The self-focusing of the fundamental pulse enhances the intensity of the third-harmonic pulse. An introduction of an upward plasma-density ramp strongly enhances the self-focusing of the fundamental laser pulse. The laser pulse attains a minimum spot size and propagates up to several
Rayleigh lengths without divergence. Due to the strong self-focusing of the fundamental laser pulse, the third-harmonic intensity enhances significantly. A considerable enhancement of the intensity of the third-harmonic is observed from the proposed mechanism. In this problem we will be discussing the generation of third-harmonic radiation of a self-focusing laser from plasma under plasma density ramp.


**ABSTRACT**

In semiconductors, free carriers are created in pairs in inter-band transitions and consist of an electron and its corresponding hole. At very high carrier densities, carrier-carrier collisions dominate over carrier-lattice collisions and carriers begin to behave collectively to form plasma. In these problems we will be discussing effect of self-focusing on resonant second harmonic generation in semiconductor.


**ABSTRACT**

In semiconductors, holes tend to be mobile and behave like electrons, though with the opposite charge, and contribute to the absorption process. The total absorption coefficient is then composed of the lattice and the carrier absorption coefficients. The latter scales with the square of the wavelength, making free carrier absorption especially relevant for infra-red illumination. In these problems we will be discussing effect of resonant third harmonic generation of a self-focusing laser in semiconductor.

5. Effect of relativistic self-focusing on third harmonic generation in a semiconductor.

**ABSTRACT**

An analysis of relativistic self-focusing of an intense laser radiation in axially inhomogeneous plasma has been also studied. The nonlinearity in the dielectric constant arises on account of the relativistic variation of mass for an arbitrary magnitude of intensity. An appropriate expression for the nonlinear dielectric constant has been used in the analysis of laser beam propagation in paraxial approximation for a circularly polarized wave. Relativistic self-focusing of laser beam in plasma, including ranges of
over densities, is of special importance for the fast ignitor laser fusion scheme. After
discussing effect of resonant third harmonic generation of a self-focusing laser in
semiconductor, we will discuss the effect of relativistic self-focusing on third harmonic
generation in a semiconductor.

**METHODOLOGY:**

1. **FORMULATION OF HYPOTHESIS**

The hypothesis for first & second problem is that we apply plasma density ramp to laser
beams in plasma to see its effect on self-focusing in the absence and in the presence of
magnetic field. Equation of beam width parameter and variation of normalized amplitude
of the second-harmonic wave $A_{2\omega}$ with normalized propagation distance $\xi$, have been
derived and will be solved numerically by using MATHEMATICA software. The results
will be plotted in ORIGIN software. For rest of the problems, we will derive the equation
of beam width parameters in semiconductor and equation of the amplitude of second/third
harmonic wave in the presence of wiggler magnetic field. We will solve these coupled
equations in MATHEMATICA and plot the results in ORIGIN.

2. **SOURCES OF DATA**

Theoretical results will be compared with experimental results collected through various
research papers.

3. **RESEARCH DESIGN:**

Theoretical analysis

**TOOLS:**

MATHEMATICA and ORIGIN software
REFERENCES


