INTRODUCTION

Graph theory is one of the most important branches of Mathematics. The term ‘graph’ was introduced by ‘Sylvester’, in his paper which was introduced in 1978 in which he has drawn an analogy between “quantic invariants” and “co-variants” of Algebra and molecular diagrams. The first paper in graph theory was published by Leonhard Euler in 1736. Later on Euler’s formula relating to the number of edges, vertices and faces if convex polyhedron was studied and generalized by Cauchy and Huillier and is at the origin of Topology (one of the important branches of Mathematics) One of the most famous and productive problems of colouring of graph is “Four colours problem” which says “Is it true that any map drawn in the plane may have its regions coloured with four colours, in such a way that any two regions having common border have different colours?” This problem remained unsolved for more than a century and then in 1969 Heinrich Heesch published a method of solving this problem using computers then a computer-aided proof produced in 1976 by Kenneth Appel and Wolfgang Haken.

Formally, given a graph $G$, a vertex labelling is a mapping which maps vertices of $G$ to a set of labels. A graph with such a function defined is called a vertex-labelled graph. Likewise, an edge labeling is a mapping from edges of $G$ to a set of "labels". In this case, $G$ is called an edge-labelled graph. There are different types of graph labelling such as Harmonious labelling, Graceful labelling, cordial labelling etc.

To define the cordial labelling of a graph $G$,
consider a vertex labelling ‘0’ and ‘1’ on G. An edge ‘ab’ will be labelled ‘0’ iff labelling of both the vertices ‘a’ and ‘b’ are same that is either ‘0’ or ‘1’ and

An edge ‘ab’ will be labelled ‘1’ iff labelling of both the vertices ‘a’ and ‘b’ are different.

Then ‘G’ is called CORDIAL iff;

\[ |\text{No. of vertices labelled ‘0’} - \text{No. of vertices labelled ‘1’}| \leq 1 \text{ and } \] 
\[ |\text{No. of edges labelled ‘0’} - \text{No. of edges labelled ‘1’}| \leq 1 \]

We are going to discuss here when a bipartite regular graph is cordial?

**Operational definition of terms and objects:**
1. **GRAPH:** Let $V$ be a nonempty set. Let

$$E^1 = \{(a,b) | a, b \in V\}$$

subset of $E^1$

The elements of $V$ are called vertices of $G$ and the elements of $E$ are called edges of $G.$

**example:** Let $V = \{1,2,3,4,5,6\}$ and $E = \{(1,2),(2,3),(3,4),(5,6)\}$ We draw a diagram which represents the above graph as follows,

![Diagram](image)

2. **Adjacent vertices:** If $e = (a,b)$ then we say $e$ is incident on $a$ (also on $b$) and $a$ & $b$ are called adjacent vertices.

3. **Degree of a vertex:** The number of edges incident on any vertex is called as degree of that vertex.

4. **Regular graph:** A graph $G$ is said to be a regular graph if degree of each vertex a same. It is called $k$-regular if degree of each vertex is $k.$

5. **Bipartite graph:** Let $G$ be a graph. If the vertex of $G$ is divided into two subsets $A$ and $B$ such that there is no edge ‘ab’ with $a, b \in A \text{ or } a, b \in B$ then $G$ is said to be bipartite that is, Every edge of $G$ joins a vertex in $A$ to a vertex in $B.$

The following graph gives a diagrammatical representation of a bipartite graph. The sets $A$ and $B$ are called partite sets of $G.$
6. Complete graph: A graph in which every vertex is adjacent to every other vertex is a complete graph. For a complete graph on n vertices degree of each vertex is n-1.

7. Complete bipartite graph: A bipartite graph G with bipartition (A,B) is said to be complete bipartite if every vertex in A is adjacent to every vertex in B and vice versa.

8. Path: If ‘a’ and ‘b’ are any two distinct vertices in G then a path from a to b is a sequence 
\[ a = a_1, a_2, ..., a_n = b \] where \( a_i \) is adjacent to \( a_{i+1} \) for \( i = 1, 2, ..., n - 1 \)

9. Cycle: A closed path is called a cycle.

10. Labelling of a graph:

   **Vertex labelling**: It is a mapping from set of vertices to set of natural numbers.

   **Edge labelling**: It is a mapping from set of edges to set of natural numbers.

11. Cordial labelling: For a given graph G label the vertices of G ‘0’ or ‘1’. And every edge ‘ab’ of G will be labeled as ‘0’ if the labeling of the vertices ‘a’ and ‘b’ are same and will be labeled as ‘1’ if the labeling of the vertices ‘a’ and ‘b’ are different. Then this labeling is called a “cordial labeling” or the graph G is called a “cordial graph” iff,

\[
|\text{number of vertices labeled } '0' - \text{number of vertices labeled } '1'| \leq 1
\]
\[
|\text{number of edges labeled } '0' - \text{number of edges labeled } '1'| \leq 1
\]