Introduction:

The origin of transportation was first presented by F.L. Hitchcock in 1941. He presented a study entitled “The Distribution of a Product from Several sources to numerous Localities”. This presentation is considered to be the first important contribution to the solution of transportation problems. In 1947 T.C. Koopmans presented in independent study, not related to Hitchcock’s, and called “Optimum Utilization of the Transportation System”. These two contributions helped in the development of transportation methods which involve a number of shipping sources and a number of destinations. The transportation problem, received this name because many of its applications involve determining how to optimally transport goods.

Transportation networks are complex, large-scale systems, and come in a variety of forms, such as road, rail, air, and waterway networks. Transportation networks provide the foundation for the functioning of our economies and societies through the movement of people, goods, and services. From an economic perspective, the supply in such network systems is represented by the underlying network topology and the cost characteristics whereas the demand is represented by the users of the transportation system. An equilibrium occurs when the number of trips between an origin (e.g., residence/ place of employment) and destination (place of employment/ residence) equals the travel demand given by the market price, typically, represented by the travel time for the trips (Nagurney (2004)).

The study of transportation networks and their efficient management dates to ancient times. It is known, for example, that Romans imposed controls over chariot traffic during different times of day in order to deal with the congestion (Banister and Button (1993)). From an economic perspective, some of the earliest
contributions to the subject date to Kohl (1841) and to Pigou (1920), who considered a two-node, two-link transportation network, identified congestion as a problem, and recognized that distinct behavioural concepts regarding route selection may prevail (see also Knight (1924)).

The formal study of transportation networks has challenged transportation scientists, economists, operations researchers, engineers, and physicists for reasons, including: the size and scope of the systems involved; the behaviour of the users of the network which may vary according to the application setting, thereby leading to different optimality/ equilibrium concepts; distinct classes of users may perceive the cost of utilizing the network in an individual fashion, and congestion, which is playing an increasing role in numerous transportation networks.

**Mathematical modelling:** Mathematical modelling is the use of mathematics to:

- Describe real world phenomena
- Investigate impartial questions about the observed world
- Explain real world phenomena
- Test ideas
- Make predictions about the real world

One can think of mathematical modelling as an activity or process that allows a mathematician to be a Chemist, an ecologist, an economist, a physiologist etc. Instead of undertaking experiments in the real world, a modeller undertakes experiments on mathematical representations of the real world.
Process of Mathematical Modelling:

There is no best model, only better models. Challenge in mathematical modelling “----not to produce the most comprehensive descriptive model but to produce the simplest possible model that incorporates the major features of the phenomenon of interest”

-Howard Emmons.

Mathematical Optimization model:-

A mathematical optimization model consists of an objective function and a set of constraints in the form of a system of equations or inequalities. Optimization models are used extensively in almost all areas of decision-making, such as engineering design and financial portfolio selection.

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Multiple objective functions: Often, the user would actually like to optimize many different objectives at ones. Usually, the different objectives are not compatible. The variables that optimize one objective may be far from optimal for the others. In practice, problems with multiple objectives are reformulated as
Mathematical optimization for transportation problem

Fundamental Decision-Making Concepts and Models: Over half a century ago, Wardrop (1952) explicitly considered alternative possible behaviors of users of transportation networks, notably, urban transportation networks and stated two principles, which are commonly named after him:

First Principle: The journey times of all actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.

Second Principle: The average journey time is minimal.

The first principle corresponds to the behavioural principle in which travellers seek to (unilaterally) determine their minimal costs of travel whereas the second principles corresponds to the behavioural principle in which the total cost in the network is minimal.

Beckmann, McGuire, and Winsten (1956) were the first to rigorously formulate these conditions mathematically. Specifically, Beckmann, McGuire, and Winstern (1956) established the equivalence between the transportation network equilibrium conditions, which state that all used paths connecting an origin/destination (O/D) pair will have equal and minimal travel times (or costs) (corresponding to Wardrop’s first principle), and the Kuhn-Tucker (1951) conditions of an appropriately constructed optimization problem, under a symmetry assumption on the underlying functions. Hence, in this case, the equilibrium link and path flows could be obtained as the solution of a mathematical programming problem. Their approach made the formulation,
analysis, and subsequent computation of solution to transportation network problems based on actual transportation networks realizable.

Dafermos and Sparrow (1969) coined the terms user-optimized (U-O) and system-optimized (S-O) transportation networks to distinguish between two distinct situations in which, respectively, users act unilaterally, in their own self-interest, in selecting their routes, and in which users select routes according to what is optimal from a societal point of view, in that the total cost in the network system is minimized. In the latter problem, marginal total costs rather than average costs are equilibrated. The former problem coincides with Wardrop’s first principle, and the latter with Wardrop’s second principle. Table 1 shows the two distinct behavioural principles for underlying transportation networks.

Table 1: Distinct Behaviour on Transportation Networks

<table>
<thead>
<tr>
<th>User-Optimization</th>
<th>System-Optimization</th>
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<tr>
<td><strong>User Equilibrium Principle</strong></td>
<td><strong>System Optimality Principle</strong></td>
</tr>
<tr>
<td>User travel costs on used paths for each O/D pair are equalized and minimal.</td>
<td>Marginal of the total travel cost on used paths for each O/D pair are equalized and minimal.</td>
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The concept of “system-optimization” is also relevant to other types of “routing models” in transportation, as well as in communications.